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MACHINERY'S DATA SHEETS

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No. 16

Mathematical Tables

PRICE 25 CENTS

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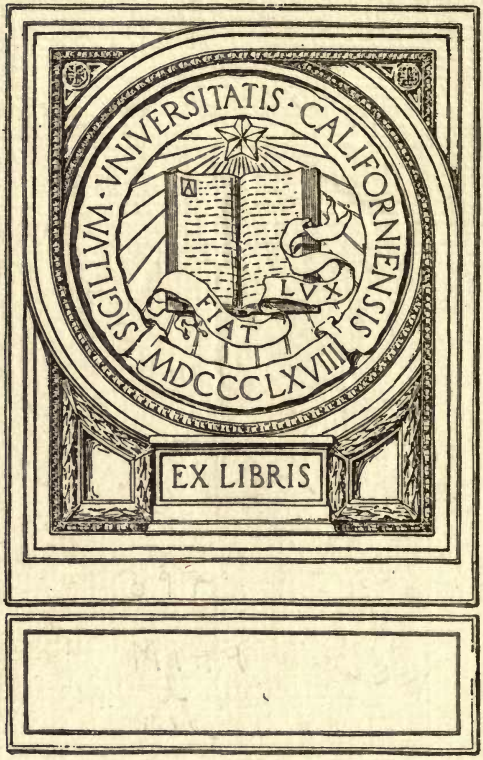
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In the following pages are compiled a number of concise mathematical tables, carefully selected from MACHINERY'S monthly Data Sheets, issued as supplements to the Engineering and Railway editions of MACHINERY since September, 1898.

In order to enhance the value of the tables, brief explanatory notes have been provided. In many cases in these notes, references are made to matter published in MACHINERY'S Reference Series, giving additional information on the subject. These references will be of considerable value to readers who wish to make a more thorough study of the subject. In a note at the foot of each table, reference is made to the page on which the explanatory note relating to the table appears.

MATHEMATICAL TABLES

Tables of Squares of Mixed Numbers

On pages 4 to 7, inclusive, are given tables of squares of mixed numbers from 1/64 to 12 by sixty-fourths. The tables are arranged in as compact a manner as possible, but a few words may be necessary to explain their use. Assume, for example, that the square of 8 5/64 is required. On page 5, 8 is located at the top of its column and 5/64 in the left-hand column. The square of 8 5/64 then is found to equal 65.256103515625. The whole number and the five first decimals are found in the column headed "8," opposite 5/64, and the seven last decimals are found in the extreme right-hand column in the table headed "End Figures." In the same way, the square of 3 3/16 is found to equal 10.16015625, as found on page 4. [MACHINERY'S Reference Series No. 52, Advanced Shop Arithmetic for the Machinist, Chapter I, Square Root.]

Functions of Fractions

On pages 8 to 11, inclusive, tables are given containing the square, cube, square root and cube root of fractions from 1/64 to 1 by sixty-fourths. The decimal equivalents of the fractions are also given, together with the logarithms for the various values. The circumferences and the areas of circles having a diameter expressed in a fraction are also given. Only so many decimals are given as are considered necessary for practical purposes. As an example of the use of the tables find the cube root of 13/32. First locate the fraction 13/32 in the left-hand column on page 9; then, in the column headed "Cube Root" and opposite 13/32, we find that the cube root equals 0.7406. In a similar manner we find that the circumference of a circle 11/64 inch in diameter is 0.54 inch, and the area of the same circle is 0.02321 square inch. [MACHINERY'S Reference Series No. 52, Advanced Shop Arithmetic for the Machinist, Chapter I, Square Root; Chapter II, Cube Root; Chapter VIII, Areas of Plane Figures; MACHINERY'S Reference Series No. 53, The Use of Logarithms and Logarithmic Tables.]

Circumferences and Diameters of Circles

On pages 12, 13 and 14, tables are given of circumferences and corresponding diameters of circles. The circumferences given include every whole number from 1 to 300. Most tables of circumferences and diameters of circles give the diameter in whole numbers and the corresponding circumference. The particular value of this table is that it gives the circumference in whole numbers and the corresponding diameter, so that the exact diameter can easily be found for any given circumference. For example, it may be required to design a pulley having a circumference of exactly 100 inches. The diameter of such a pulley, as found from page 12, would be 31.831 inches. [MACHINERY'S Reference Series No. 52, Advanced Shop Arithmetic for the Machinist, Chapter VIII, Areas of Plane Figures.]

Lengths of Chords for Spacing Circles

On page 16 is given a table of lengths of chords for spacing off circles. The object of this table is to make possible the division of the periphery of a circle into a number of equal parts, without having to resort to complicated calcula-

(Continued on page 15.)

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TABLE OF SQUARES OF MIXED NUMBERS FROM 1-64 TO 12 BY 1-64THS.

	0	1	2	3	4	5	End Figures.
1/64	.00024	1.03149	4.06274	9.09399	16.12524	25.15649	4140625
1/32	.0009	1.0634	4.1259	9.1884	16.2509	25.3134	765625
3/64	.00219	1.09594	4.18969	9.28344	16.37719	25.47094	7265625
1/16	.003	1.128	4.253	9.378	16.503	25.628	90625
5/64	.00610	1.16235	4.31860	9.47485	16.63110	25.78735	3515625
3/32	.0087	1.1962	4.3837	9.5712	16.7587	25.9462	890625
7/64	.01196	1.23071	4.44946	9.66821	16.88696	26.10571	2890625
1/8	.01	1.26	4.51	9.76	17.01	26.26	5625
9/64	.01977	1.30102	4.58227	9.86352	17.14477	26.42602	5390625
5/32	.0244	1.3369	4.6494	9.9619	17.2744	26.5869	140625
11/64	.02954	1.37329	4.71704	10.06079	17.40454	26.74829	1015625
3/16	.035	1.410	4.785	10.160	17.535	26.910	15625
13/64	.04125	1.44750	4.85375	10.26000	17.66625	27.07250	9765625
7/32	.0473	1.4853	4.9228	10.3603	17.7978	27.2353	515625
15/64	.05493	1.52368	4.99243	10.46118	17.92993	27.39868	1640625
1/4	.0	1.5	5.0	10.5	18.0	27.5	625
17/64	.07055	1.60180	5.13305	10.66430	18.19555	27.72680	6640625
9/32	.0791	1.6416	5.2041	10.7666	18.3291	27.8916	015625
19/64	.08813	1.68188	5.27563	10.86938	18.46313	28.05688	4765625
5/16	.097	1.722	5.347	10.972	18.597	28.222	65625
21/64	.10766	1.76391	5.42016	11.07641	18.73266	28.38891	6015625
11/32	.1181	1.8056	5.4931	11.1806	18.8681	28.5556	640625
23/64	.12915	1.84790	5.56665	11.28540	19.00415	28.72290	0390625
3/8	.14	1.89	5.64	11.39	19.14	28.89	0625
25/64	.15258	1.93383	5.71508	11.49633	19.27758	29.05883	7890625
13/32	.1650	1.9775	5.7900	11.6025	19.4150	29.2275	390625
27/64	.17797	2.02172	5.86547	11.70922	19.55297	29.39672	8515625
7/16	.191	2.066	5.941	11.816	19.691	29.566	40625
29/64	.20532	2.11157	6.01782	11.92407	19.83032	29.73657	2265625
15/32	.2197	2.1572	6.0947	12.0322	19.9697	29.9072	265625
31/64	.23461	2.20336	6.17211	12.14086	20.10961	30.07836	9140625

TABLE OF SQUARES OF MIXED NUMBERS. (Continued.)

	6	7	8	9	10	11	End Figures.
1/64	36.18774	49.21899	64.25024	81.28149	100.31274	121.34399	4140625
1/32	36.3759	49.4384	64.5009	81.5634	100.6259	121.6884	765625
3/64	36.56469	49.65844	64.75219	81.84594	100.93969	122.03344	7265625
1/16	36.753	49.878	65.003	82.128	101.253	122.378	90625
5/64	36.94360	50.09985	65.25610	82.41235	101.56860	122.72485	3515625
3/32	37.1337	50.3212	65.5087	82.6962	101.8837	123.0712	890625
7/64	37.32446	50.54321	65.76196	82.98071	102.19946	123.41821	2890625
1/8	37.51	50.76	66.01	83.26	102.51	123.76	5625
9/64	37.70727	50.98852	66.26977	83.55102	102.83227	124.11352	5390625
5/32	37.8994	51.2119	66.5244	83.8369	103.1494	124.4619	140625
11/64	38.09204	51.43579	66.77954	84.12329	103.46704	124.81079	1015625
3/16	38.285	51.660	67.035	84.410	103.785	125.160	15625
13/64	38.47875	51.88500	67.29125	84.69750	104.10375	125.51000	9765625
7/32	38.6728	52.1103	67.5478	84.9853	104.4228	125.8603	515625
15/64	38.86743	52.33618	67.80493	85.27368	104.74243	126.21118	1640625
1/4	39.0	52.5	68.0	85.5	105.0	126.5	625
17/64	39.25805	52.78930	68.32055	85.85180	105.38305	126.91430	6640625
9/32	39.4541	53.0166	68.5791	86.1416	105.7041	127.2666	015625
19/64	39.65063	53.24438	68.83813	86.43188	106.02563	127.61938	4765625
5/16	39.847	53.472	69.097	86.722	106.347	127.972	65625
21/64	40.04516	53.70141	69.35766	87.01391	106.67016	128.32641	6015625
11/32	40.2431	53.9306	69.6181	87.3056	106.9931	128.6806	640625
23/64	40.44165	54.16040	69.87915	87.59790	107.31665	129.03540	0390625
3/8	40.64	54.39	70.14	87.89	107.64	129.39	0625
25/64	40.84008	54.62133	70.40258	88.18383	107.96508	129.74633	7890625
13/32	41.0400	54.8525	70.6650	88.4775	108.2900	130.1025	390625
27/64	41.24047	55.08422	70.92797	88.77172	108.61547	130.45922	8515625
7/16	41.441	55.316	71.191	89.066	108.941	130.816	40625
29/64	41.64282	55.54907	71.45532	89.36157	109.26782	131.17407	2265625
15/32	41.8447	55.7822	71.7197	89.6572	109.5947	131.5322	265625
31/64	42.04711	56.01586	71.98461	89.95336	109.92211	131.89086	9140625

TABLE OF SQUARES OF MIXED NUMBERS. (Continued.)

	0	1	2	3	4	5	End Figures.
1/8	.25	2.25	6.25	12.25	20.25	30.25	00
33/64	.26586	2.29711	6.32836	12.35961	20.39086	30.42211	9140625
17/32	.2822	2.3447	6.4072	12.4697	20.5322	30.5947	265625
35/64	.29907	2.39282	6.48657	12.58032	20.67407	30.76782	2265625
9/16	.316	2.441	6.566	12.691	20.816	30.941	40625
37/64	.33422	2.49047	6.64672	12.80297	20.95922	31.11547	8515625
19/32	.3525	2.5400	6.7275	12.9150	21.1025	31.2900	390625
39/64	.37133	2.59008	6.80883	13.02758	21.24633	31.46508	7890625
5/8	.39	2.64	6.89	13.14	21.39	31.64	0625
41/64	.41040	2.69165	6.97290	13.25415	21.53540	31.81665	0390625
21/32	.4306	2.7431	7.0556	13.3681	21.6806	31.9931	640625
43/64	.45141	2.79516	7.13891	13.48266	21.82641	32.17016	6015625
11/16	.472	2.847	7.222	13.597	21.972	32.347	65625
45/64	.49438	2.90063	7.30688	13.71313	22.11938	32.52563	4765625
23/32	.5166	2.9541	7.3916	13.8291	22.2666	32.7041	015625
47/64	.53930	3.00805	7.47680	13.94555	22.41430	32.88305	6640625
3/4	.5	3.0	7.5	14.0	22.5	33.0	625
49/64	.58618	3.11743	7.64868	14.17993	22.71118	33.24243	1640625
25/32	.6103	3.1728	7.7353	14.2978	22.8603	33.4228	515625
51/64	.63500	3.22875	7.82250	14.41625	23.01000	33.60375	9765625
13/16	.660	3.285	7.910	14.535	23.160	33.785	15625
53/64	.68579	3.34204	7.99829	14.65454	23.31079	33.96704	1015625
27/32	.7119	3.3994	8.0869	14.7744	23.4619	34.1494	140625
55/64	.73852	3.45727	8.17602	14.89477	23.61352	34.33227	5390625
7/8	.76	3.51	8.26	15.01	23.76	34.51	5625
57/64	.79321	3.57446	8.35571	15.13696	23.91821	34.69946	2890625
29/32	.8212	3.6337	8.4462	15.2587	24.0712	34.8837	890625
59/64	.84985	3.69360	8.53735	15.38110	24.22485	35.06860	3515625
15/16	.878	3.753	8.628	15.503	24.378	35.253	90625
61/64	.90844	3.81469	8.72094	15.62719	24.53344	35.43969	7265625
31/32	.9384	3.8759	8.8134	15.7509	24.6884	35.6259	765625
63/64	.96899	3.93774	8.90649	15.87524	24.84399	35.81274	4140625

Contributed by G. R. Carothers, MACHINERY'S Data Sheet No. 50. Explanatory note: Page 3.

TABLE OF SQUARES OF MIXED NUMBERS. (Continued.)

	6	7	8	9	10	11	End Figures.
1/2	42.25	56.25	72.25	90.25	110.25	132.25	000
33/64	42.45336	56.48461	72.51586	90.54711	110.57836	132.60961	9140625
17/32	42.6572	56.7197	72.7822	90.8447	110.9072	132.9697	265625
35/64	42.86157	56.95532	73.04907	91.14282	111.23657	133.33032	2265625
9/16	43.066	57.191	73.316	91.441	111.566	133.691	40625
37/64	43.27172	57.42797	73.58422	91.74047	111.89672	134.05297	8515625
19/32	43.4775	57.6650	73.8525	92.0400	112.2275	134.4150	390625
39/64	43.68383	57.90258	74.12133	92.34008	112.55883	134.77758	7890625
5/8	43.89	58.14	74.39	92.64	112.89	135.14	0625
41/64	44.09790	58.37915	74.66040	92.94165	113.22290	135.50415	0390625
21/32	44.3056	58.6181	74.9306	93.2431	113.5556	135.8681	640625
43/64	44.51391	58.85766	75.20141	93.54516	113.88891	136.23266	6015625
11/16	44.722	59.097	75.472	93.847	114.222	136.597	65625
45/64	44.93188	59.33813	75.74438	94.15063	114.55688	136.96313	4765625
23/32	45.1416	59.5791	76.0166	94.4541	114.8916	137.3291	015625
47/64	45.35180	59.82055	76.28930	94.75805	115.22680	137.69555	6640625
3/4	45.5	60.0	76.5	95.0	115.5	138.0	625
49/64	45.77368	60.30493	76.83618	95.36743	115.89868	138.42993	1640625
25/32	45.9853	60.5478	77.1103	95.6728	116.2353	138.7978	515625
51/64	46.19750	60.79125	77.38500	95.97875	116.57250	139.16625	9765625
13/16	46.410	61.035	77.660	96.285	116.910	139.535	15625
53/64	46.62329	61.27954	77.93579	96.59204	117.24829	139.90454	1015625
27/32	46.8369	61.5244	78.2119	96.8994	117.5869	140.2744	140625
55/64	47.05102	61.76977	78.48852	97.20727	117.92602	140.64477	5390625
7/8	47.26	62.01	78.76	97.51	118.26	141.01	5625
57/64	47.48071	62.26190	79.04321	97.82446	118.60571	141.38696	2890625
29/32	47.6962	62.5087	79.3212	98.1337	118.9462	141.7587	890625
59/64	47.91235	62.75610	79.59985	98.44360	119.28735	142.13110	3515625
15/16	48.128	63.003	79.878	98.753	119.628	142.503	90625
61/64	48.34594	63.25219	80.15844	99.06469	119.97094	142.87719	7265625
31/32	48.5634	63.5009	80.4384	99.3759	120.3134	143.2509	765625
63/64	48.78149	63.75024	80.71899	99.68774	120.65649	143.62524	4140625

TABLE OF FUNCTIONS OF FRACTIONS FROM 1/64 TO 1 BY 64THS

FRACTION	DEC. EQUIV.	LOG.	SQUARE	LOG.	CUBE	LOG.	SQ.ROOT	LOG.	CUBE ROOT	LOG.	CIRCUMFERENCE	LOG.	AREA CIRCLE	LOG.
1/64	.015625	$\bar{2}$.19382	.000244	$\bar{4}$.38764	.000003815	$\bar{6}$.58146	.1250	$\bar{1}$.09691	.2500	$\bar{1}$.39794	.04909	$\bar{2}$.69097	.000192	$\bar{4}$.28273
1/32	.03125	$\bar{2}$.49485	.0009765	$\bar{4}$.98970	.00003052	$\bar{5}$.48455	.1768	$\bar{1}$.24743	.3150	$\bar{1}$.49828	.09818	$\bar{2}$.99200	.000767	$\bar{4}$.88479
3/64	.046875	$\bar{2}$.67094	.002197	$\bar{3}$.34188	.000103	$\bar{4}$.01282	.2165	$\bar{1}$.33547	.3606	$\bar{1}$.55698	.1473	$\bar{1}$.16809	.001726	$\bar{3}$.23697
1/16	.0625	$\bar{2}$.79588	.003906	$\bar{3}$.59176	.0002442	$\bar{4}$.38764	.25000	$\bar{1}$.39794	.39685	$\bar{1}$.59863	.1963	$\bar{1}$.29303	.003068	$\bar{3}$.48685
5/64	.078125	$\bar{2}$.89279	.006104	$\bar{3}$.78558	.0004768	$\bar{4}$.67837	.2795	$\bar{1}$.44639	.4275	$\bar{1}$.63093	.2455	$\bar{1}$.38994	.004794	$\bar{3}$.68067
3/32	.09375	$\bar{2}$.97197	.008789	$\bar{3}$.94394	.0008240	$\bar{4}$.91591	.3062	$\bar{1}$.48598	.4543	$\bar{1}$.65732	.2945	$\bar{1}$.46912	.006903	$\bar{3}$.83903
7/64	.109375	$\bar{1}$.03892	.01196	$\bar{2}$.07784	.001308	$\bar{3}$.11676	.3307	$\bar{1}$.51946	.4782	$\bar{1}$.67964	.3436	$\bar{1}$.53607	.009396	$\bar{3}$.97296
1/8	.1250	$\bar{1}$.09691	.01563	$\bar{2}$.19382	.001953	$\bar{3}$.29073	.3535	$\bar{1}$.54845	.5000	$\bar{1}$.69897	.3927	$\bar{1}$.59406	.01228	$\bar{2}$.08891
9/64	.140625	$\bar{1}$.14807	.01978	$\bar{2}$.29614	.002781	$\bar{3}$.44421	.3750	$\bar{1}$.57403	.5200	$\bar{1}$.71602	.4438	$\bar{1}$.64722	.01553	$\bar{2}$.19123
5/32	.15625	$\bar{1}$.19382	.02441	$\bar{2}$.38764	.003815	$\bar{3}$.58146	.3953	$\bar{1}$.59691	.5386	$\bar{1}$.73127	.4909	$\bar{1}$.69097	.01916	$\bar{2}$.28273
11/64	.171875	$\bar{1}$.23522	.02954	$\bar{2}$.47044	.005078	$\bar{3}$.70566	.4161	$\bar{1}$.61761	.5560	$\bar{1}$.74507	.5400	$\bar{1}$.73237	.02321	$\bar{2}$.36568
3/16	.1875	$\bar{1}$.27300	.03516	$\bar{2}$.54600	.006592	$\bar{3}$.81900	.4330	$\bar{1}$.63650	.5724	$\bar{1}$.75767	.5890	$\bar{1}$.77015	.02761	$\bar{2}$.44109
13/64	.203125	$\bar{1}$.30776	.04126	$\bar{2}$.61553	.008381	$\bar{3}$.92329	.4507	$\bar{1}$.65388	.5878	$\bar{1}$.76925	.6381	$\bar{1}$.80491	03241	$\bar{2}$.51061
7/32	.21875	$\bar{1}$.33995	.04786	$\bar{2}$.67980	.01047	$\bar{2}$.01985	.4677	$\bar{1}$.66998	.6025	$\bar{1}$.77998	.6872	$\bar{1}$.83710	.03758	$\bar{2}$.57499
15/64	.234375	$\bar{1}$.36992	.05498	$\bar{2}$.73984	.01287	$\bar{2}$.10976	.4841	$\bar{1}$.68496	.6166	$\bar{1}$.78997	.7363	$\bar{1}$.86707	.04314	$\bar{2}$.63498
1/4	.2500	$\bar{1}$.39794	.0625	$\bar{2}$.79588	.01562	$\bar{2}$.19382	.500	$\bar{1}$.69897	.6300	$\bar{1}$.79961	.7854	$\bar{1}$.89509	.04909	$\bar{2}$.69097

Contributed by Robert S. Brown, MACHINERY'S Data Sheet No. 48. Explanatory note: Page 3.

TABLE OF FUNCTIONS OF FRACTIONS (CONTINUED)

FRACTION	DEC. EQUIV.	LOG.	SQUARE	LOG.	CUBE	LOG.	SQ. ROOT	LOG.	CUBE ROOT	LOG.	CIRCUMFERENCE	LOG.	AREA CIRCLE	LOG.
17/64	.265625	$\bar{1}$.42427	.07056	$\bar{2}$.84854	.01874	$\bar{2}$.27281	.5154	$\bar{1}$.71213	.6428	$\bar{1}$.80809	.8345	$\bar{1}$.92142	.05541	$\bar{2}$.74363
9/32	.28125	$\bar{1}$.44910	.07910	$\bar{1}$.89820	.02225	$\bar{2}$.34730	.5303	$\bar{1}$.72455	.6552	$\bar{1}$.81636	.8836	$\bar{1}$.94625	.06213	$\bar{2}$.79329
19/64	.296875	$\bar{1}$.47258	08813	$\bar{2}$.94516	.02616	$\bar{2}$.41774	.5449	$\bar{1}$.73629	.6671	$\bar{1}$.82419	.9327	$\bar{1}$.96973	.06922	$\bar{2}$.84025
5/16	.3125	$\bar{1}$.49485	.09766	$\bar{2}$.98970	.03052	$\bar{2}$.48455	5590	$\bar{1}$.74742	.6786	$\bar{1}$.83161	.9817	$\bar{1}$.99200	.07670	$\bar{2}$.88479
21/64	.328125	$\bar{1}$.51604	.1077	$\bar{1}$.03208	.03533	$\bar{2}$.54812	.5728	$\bar{1}$.75802	.6897	$\bar{1}$.83868	1.031	.01319	.08456	$\bar{2}$.92717
11/32	.34375	$\bar{1}$.53625	.1182	$\bar{1}$.07250	.04062	$\bar{2}$.60875	.5863	$\bar{1}$.76812	.7005	$\bar{1}$.84541	1.080	.03340	.09281	$\bar{2}$.96759
23/32	.359375	$\bar{1}$.55555	.12913	$\bar{1}$.11110	.04641	$\bar{2}$.66665	.5995	$\bar{1}$.77777	.71106	$\bar{1}$.85185	1.129	.05270	.1014	$\bar{1}$.00619
3/8	.3750	$\bar{1}$.57403	.1406	$\bar{1}$.14806	.05273	$\bar{2}$.72209	.6124	$\bar{1}$.78701	.7211	$\bar{1}$.85801	1.178	.07118	.1104	$\bar{1}$.04315
25/64	.390625	$\bar{1}$.59176	.1526	$\bar{1}$.18352	.05960	$\bar{2}$.77528	.6250	$\bar{1}$.79588	.7310	$\bar{1}$.86392	1.227	.08841	.1226	$\bar{1}$.07861
13/32	.40625	$\bar{1}$.60879	.1650	$\bar{1}$.21758	.06705	$\bar{2}$.82637	.6374	$\bar{1}$.80439	.7406	$\bar{1}$.86959	1.276	.10594	.1296	$\bar{1}$.11267
27/64	.421875	$\bar{1}$.62519	.17800	$\bar{1}$.25036	.07508	$\bar{2}$.87554	.6495	$\bar{1}$.81259	.75000	$\bar{1}$.87506	1.325	.12233	.1398	$\bar{1}$.14545
7/16	.4375	$\bar{1}$.64098	.1914	$\bar{1}$.28196	.08374	$\bar{2}$.92294	.6614	$\bar{1}$.82049	.7592	$\bar{1}$.88032	1.374	.13813	.1503	$\bar{1}$.17705
29/64	.453125	$\bar{1}$.65622	.2053	$\bar{1}$.31244	.09304	$\bar{2}$.96866	.6732	$\bar{1}$.82811	.7681	$\bar{1}$.88540	1.424	.15337	.1613	$\bar{1}$.20753
15/32	.46875	$\bar{1}$.67094	.2197	$\bar{1}$.34188	.1030	$\bar{1}$.01282	.6847	$\bar{1}$.83547	.7768	$\bar{1}$.89031	1.473	.16809	.1726	$\bar{1}$.23697
31/64	.484375	$\bar{1}$.68518	.2346	$\bar{1}$.37036	.1136	$\bar{1}$.05554	.6960	$\bar{1}$.84259	.7853	$\bar{1}$.89506	1.522	.18233	.1843	$\bar{1}$.26545
1/2	.5000	$\bar{1}$.69897	.2500	$\bar{1}$.39794	.1250	$\bar{1}$.09691	.7071	$\bar{1}$.84948	.7937	$\bar{1}$.89966	1.571	.19612	.1963	$\bar{1}$.29303

TABLE OF FUNCTIONS OF FRACTIONS (CONTINUED)

FRACTION	DEC. EQUIV.	LOG.	SQUARE	LOG.	CUBE	LOG.	SQ. ROOT	LOG.	CUBE ROOT	LOG.	CIRCUMFERENCE	LOG.	AREA CIRCLE	LOG.
33/64	.515625	$\bar{1}$.71233	.26595	$\bar{1}$.42466	.1371	$\bar{1}$.13699	.7181	$\bar{1}$.85616	.8019	$\bar{1}$.90411	1.620	.20948	.2088	$\bar{1}$.31975
17/32	.53125	$\bar{1}$.72530	.2822	$\bar{1}$.45060	.1499	$\bar{1}$.17590	.7289	$\bar{1}$.86265	.8099	$\bar{1}$.90843	1.669	.22245	.2217	$\bar{1}$.34569
35/64	.546875	$\bar{1}$.73789	.2991	$\bar{1}$.47578	.1636	$\bar{1}$.21367	.7395	$\bar{1}$.86894	.8178	$\bar{1}$.91263	1.718	.23504	.2349	$\bar{1}$.37087
9/16	.5625	$\bar{1}$.75012	.3164	$\bar{1}$.50024	.1780	$\bar{1}$.25036	.7500	$\bar{1}$.87506	.8255	$\bar{1}$.91670	1.767	.24727	.2485	$\bar{1}$.39553
37/64	.578125	$\bar{1}$.76202	.3342	$\bar{1}$.52404	.1932	$\bar{1}$.28606	.7603	$\bar{1}$.88101	.8331	$\bar{1}$.92067	1.816	.25917	.2625	$\bar{1}$.41913
19/32	.59375	$\bar{1}$.77361	.3525	$\bar{1}$.54722	.2093	$\bar{1}$.32083	.7706	$\bar{1}$.88680	.8405	$\bar{1}$.92453	1.865	.27076	.2769	$\bar{1}$.44231
39/64	.609375	$\bar{1}$.78488	.3713	$\bar{1}$.56976	.2263	$\bar{1}$.35464	.7806	$\bar{1}$.89244	.8478	$\bar{1}$.92829	1.914	.28203	.2916	$\bar{1}$.46485
5/8	.6250	$\bar{1}$.79588	.3906	$\bar{1}$.59176	.2441	$\bar{1}$.38764	.7906	$\bar{1}$.89794	.8550	$\bar{1}$.93196	1.963	.29603	.3068	$\bar{1}$.48685
41/64	.640625	$\bar{1}$.80661	.4104	$\bar{1}$.61322	.2629	$\bar{1}$.41963	.8004	$\bar{1}$.90330	.8621	$\bar{1}$.93553	2.013	.30376	.3223	$\bar{1}$.50831
21/32	.65625	$\bar{1}$.81707	.4307	$\bar{1}$.63414	.2826	$\bar{1}$.45121	.8101	$\bar{1}$.90853	.8690	$\bar{1}$.93902	2.062	.31422	.3382	$\bar{1}$.52923
43/64	.671875	$\bar{1}$.82729	.4514	$\bar{1}$.65458	.3033	$\bar{1}$.48187	.8197	$\bar{1}$.91364	.8758	$\bar{1}$.94243	2.111	.32444	.3545	$\bar{1}$.54967
11/16	.6875	$\bar{1}$.83727	.4727	$\bar{1}$.67454	.3250	$\bar{1}$.51181	.8292	$\bar{1}$.91864	.8826	$\bar{1}$.94575	2.160	.33442	.3712	$\bar{1}$.56963
45/64	.703125	$\bar{1}$.84704	.4944	$\bar{1}$.69408	.3476	$\bar{1}$.54112	.8385	$\bar{1}$.92352	.8892	$\bar{1}$.94901	2.209	.34419	.3883	$\bar{1}$.58917
23/32	.71875	$\bar{1}$.85658	.5166	$\bar{1}$.71316	.3713	$\bar{1}$.56974	.8478	$\bar{1}$.92829	.8958	$\bar{1}$.95219	2.258	.35373	.4057	$\bar{1}$.60825
47/64	.734375	$\bar{1}$.86592	.5398	$\bar{1}$.73184	.3961	$\bar{1}$.59776	.8569	$\bar{1}$.93296	.9022	$\bar{1}$.95530	2.307	.36307	.4236	$\bar{1}$.62693
3/4	.7500	$\bar{1}$.87506	5625	$\bar{1}$.75012	.4219	$\bar{1}$.62518	.8660	$\bar{1}$.93753	.9086	$\bar{1}$.95835	2.356	.37221	.4418	$\bar{1}$.64521

Contributed by Robert S. Brown, MACHINERY's Data Sheet No. 48. Explanatory note: Page 3.

TABLE OF FUNCTIONS OF FRACTIONS (CONTINUED)

FRACTION	DEC. EQUIV.	LOG.	SQUARE	LOG.	CUBE	LOG.	SQ. ROOT	LOG.	CUBE ROOT	LOG.	CIRCUMFERENCE	LOG.	AREA CIRCLE	LOG.
49/64	.765625	$\bar{1}$.88402	.5862	$\bar{1}$.76804	.4488	$\bar{1}$.65206	.8750	$\bar{1}$.94201	.9148	$\bar{1}$.96134	2.405	.38117	.4604	$\bar{1}$.66313
25/32	.78125	$\bar{1}$.89279	.6104	$\bar{1}$.78558	.4768	$\bar{1}$.67837	.8839	$\bar{1}$.94640	.9210	$\bar{1}$.96426	2.454	.38994	4794	$\bar{1}$.68067
51/64	.796875	$\bar{1}$.90139	.6350	$\bar{1}$.80278	.5060	$\bar{1}$.70417	.8927	$\bar{1}$.95069	.9271	$\bar{1}$.96713	2.503	.39854	.4987	$\bar{1}$.69781
13/16	.8125	$\bar{1}$.90982	.6602	$\bar{1}$.81964	.5364	$\bar{1}$.72946	.9014	$\bar{1}$.95491	.9331	$\bar{1}$.96994	2.553	.40697	.5185	$\bar{1}$.71473
53/64	.828125	$\bar{1}$.91810	.6858	$\bar{1}$.83620	.5679	$\bar{1}$.75430	.91001	$\bar{1}$.95905	.9391	$\bar{1}$.97270	2.602	.41525	.5386	$\bar{1}$.73129
27/32	.84375	$\bar{1}$.92622	.7119	$\bar{1}$.85244	.6007	$\bar{1}$.77866	.9186	$\bar{1}$.96311	.9449	$\bar{1}$.97540	2.651	.42337	.5592	$\bar{1}$.74753
55/64	.859375	$\bar{1}$.93419	.7385	$\bar{1}$.86838	.6347	$\bar{1}$.80257	.9270	$\bar{1}$.96709	.9507	$\bar{1}$.97806	2.700	.43134	.5801	$\bar{1}$.76347
7/8	.8750	$\bar{1}$.94201	.7656	$\bar{1}$.88402	.6699	$\bar{1}$.82603	.9354	$\bar{1}$.97101	.9565	$\bar{1}$.98067	2.749	.43916	.6013	$\bar{1}$.77911
57/64	.890625	$\bar{1}$.94969	.7932	$\bar{1}$.89938	.7064	$\bar{1}$.84907	.9437	$\bar{1}$.97484	.9621	$\bar{1}$.98323	2.798	.44684	.6230	$\bar{1}$.79447
29/32	.90625	$\bar{1}$.95725	.8213	$\bar{1}$.91450	.7443	$\bar{1}$.87175	.9520	$\bar{1}$.97862	.9677	$\bar{1}$.98575	2.847	.45440	.6450	$\bar{1}$.80959
59/64	.921875	$\bar{1}$.96467	.8499	$\bar{1}$.92934	.7835	$\bar{1}$.89401	.9601	$\bar{1}$.98233	.9732	$\bar{1}$.98822	2.896	.46182	.6675	$\bar{1}$.82443
15/16	.9375	$\bar{1}$.97197	.8789	$\bar{1}$.94394	.8240	$\bar{1}$.91591	.9682	$\bar{1}$.98598	.9787	$\bar{1}$.99065	2.945	.46912	.6903	$\bar{1}$.83903
61/64	.953125	$\bar{1}$.97915	.9084	$\bar{1}$.95830	.8659	$\bar{1}$.93745	.9763	$\bar{1}$.98957	.9841	$\bar{1}$.99305	2.994	.47630	.7135	$\bar{1}$.86339
31/32	.96875	$\bar{1}$.98621	.9385	$\bar{1}$.97242	.9091	$\bar{1}$.95863	.9843	$\bar{1}$.99310	.9895	$\bar{1}$.99540	3.043	.48336	.7371	$\bar{1}$.86751
63/64	.984375	$\bar{1}$.99316	.9690	$\bar{1}$.98632	.9539	$\bar{1}$.97948	.9922	$\bar{1}$.99658	.9948	$\bar{1}$.99772	3.093	.49031	.7610	$\bar{1}$.88141
1	1	0	1	0	1	0	1	0	1	0	3.1416	.49715	.7854	$\bar{1}$.89509

CIRCUMFERENCES AND DIAMETERS OF CIRCLES.

Circum.	Diameter.	Circum.	Diameter.	Circum.	Diameter.	Circum.	Diameter.
1	.3183	26	8.2761	51	16.2338	76	24.1916
2	.6366	27	8.5944	52	16.5521	77	24.5099
3	.9549	28	8.9127	53	16.8704	78	24.8282
4	1.2732	29	9.2310	54	17.1887	79	25.1465
5	1.5915	30	9.5493	55	17.5070	80	25.4648
6	1.9099	31	9.8676	56	17.8254	81	25.7831
7	2.2282	32	10.1859	57	18.1437	82	26.1014
8	2.5465	33	10.5042	58	18.4620	83	26.4197
9	2.8648	34	10.8225	59	18.7803	84	26.7380
10	3.1831	35	11.1408	60	19.0986	85	27.0563
11	3.5014	36	11.4592	61	19.4169	86	27.3747
12	3.8197	37	11.7775	62	19.7352	87	27.6930
13	4.1380	38	12.0958	63	20.0535	88	28.0113
14	4.4563	39	12.4141	64	20.3718	89	28.3296
15	4.7746	40	12.7324	65	20.6901	90	28.6479
16	5.0930	41	13.0507	66	21.0085	91	28.9662
17	5.4113	42	13.3690	67	21.3268	92	29.2845
18	5.7296	43	13.6873	68	21.6451	93	29.6028
19	6.0479	44	14.0056	69	21.9634	94	29.9211
20	6.3662	45	14.3239	70	22.2817	95	30.2394
21	6.6845	46	14.6423	71	22.6000	96	30.5577
22	7.0028	47	14.9606	72	22.9183	97	30.8761
23	7.3211	48	15.2789	73	23.2366	98	31.1944
24	7.6394	49	15.5972	74	23.5549	99	31.5127
25	7.9577	50	15.9155	75	23.8732	100	31.8310

CIRCUMFERENCES AND DIAMETERS OF CIRCLES (Continued).

Circum.	Diameter.	Circum.	Diameter.	Circum.	Diameter.	Circum.	Diameter.
101	32.1493	126	40.1070	151	48.0648	176	56.0225
102	32.4676	127	40.4254	152	48.3831	177	56.3408
103	32.7859	128	40.7437	153	48.7014	178	56.6592
104	33.1042	129	41.0620	154	49.0197	179	56.9775
105	33.4225	130	41.3803	155	49.3380	180	57.2958
106	33.7408	131	41.6986	156	49.6563	181	57.6141
107	34.0592	132	42.0169	157	49.9747	182	57.9324
108	34.3775	133	42.3352	158	50.2930	183	58.2507
109	34.6958	134	42.6535	159	50.6113	184	58.5690
110	35.0141	135	42.9718	160	50.9296	185	58.8873
111	35.3324	136	43.2901	161	51.2479	186	59.2056
112	35.6507	137	43.6085	162	51.5662	187	59.5239
113	35.9690	138	43.9268	163	51.8845	188	59.8423
114	36.2873	139	44.2451	164	52.2028	189	60.1606
115	36.6056	140	44.5634	165	52.5211	190	60.4789
116	36.9239	141	44.8817	166	52.8394	191	60.7972
117	37.2423	142	45.2000	167	53.1578	192	61.1155
118	37.5606	143	45.5183	168	53.4761	193	61.4338
119	37.8789	144	45.8366	169	53.7944	194	61.7521
120	38.1972	145	46.1549	170	54.1127	195	62.0704
121	38.5155	146	46.4732	171	54.4310	196	62.3887
122	38.8338	147	46.7916	172	54.7493	197	62.7070
123	39.1521	148	47.1099	173	55.0676	198	63.0254
124	39.4704	149	47.4282	174	55.3859	199	63.3437
125	39.7887	150	47.7465	175	55.7042	200	63.6620

CIRCUMFERENCES AND DIAMETERS OF CIRCLES (Continued).

Circum.	Diameter.	Circum.	Diameter.	Circum.	Diameter.	Circum.	Diameter.
201	63.9803	226	71.9380	251	79.8958	276	87.8535
202	64.2986	227	72.2563	252	80.2141	277	88.1718
203	64.6169	228	72.5747	253	80.5324	278	88.4901
204	64.9352	229	72.8930	254	80.8507	279	88.8085
205	65.2535	230	73.2113	255	81.1690	280	89.1268
206	65.5718	231	73.5296	256	81.4873	281	89.4451
207	65.8901	232	73.8479	257	81.8056	282	89.7634
208	66.2085	233	74.1662	258	82.1240	283	90.0817
209	66.5268	234	74.4845	259	82.4423	284	90.4000
210	66.8451	235	74.8028	260	82.7606	285	90.7183
211	67.1634	236	75.1211	261	83.0789	286	91.0366
212	67.4817	237	75.4394	262	83.3972	287	91.3549
213	67.8000	238	75.7578	263	83.7155	288	91.6732
214	68.1183	239	76.0761	264	84.0338	289	91.9916
215	68.4366	240	76.3944	265	84.3521	290	92.3099
216	68.7549	241	76.7127	266	84.6704	291	92.6282
217	69.0732	242	77.0310	267	84.9887	292	92.9465
218	69.3916	243	77.3493	268	85.3070	293	93.2648
219	69.7099	244	77.6676	269	85.6254	294	93.5831
220	70.0282	245	77.9859	270	85.9437	295	93.9014
221	70.3465	246	78.3042	271	86.2620	296	94.2197
222	70.6648	247	78.6225	272	86.5803	297	94.5380
223	70.9831	248	78.9409	273	86.8986	298	94.8563
224	71.3014	249	79.2592	274	87.2169	299	95.1747
225	71.6197	250	79.5775	275	87.5352	300	95.4930

tions. The table is calculated for circles having a diameter equal to 1. For circles of other diameters, the length of chord given in the table should be multiplied by the diameter of the circle. For example, assume that it is required to divide the periphery of a circle of 20 inches diameter into thirty-two equal parts. From the table we find that the length of the chord would be 0.098 inch, if the diameter of the circle were one inch. In this case, where the diameter is 20 inches, the length of the chord for one division will be $20 \times 0.098 = 1.96$ inch.

On pages 18 and 19 is given an additional table for the spacing off of circles, the table in this case being worked out for diameters from 1/16 inch up to 14 inches. As an example, assume that we wish to divide a circle having a diameter of 6½ inches into seven equal parts. From the table on page 19 we find in the column headed "6" and in line with 7 divisions, as located in the left-hand column, that the length of the chord for a 6-inch circle is 2.604 inches. From page 18 we find that the length of the chord for a half-inch diameter circle is 0.217 inch. The sum of these two values, $2.604 + 0.217 = 2.821$ inches, is the length of the chord required for spacing off a circle, 6½ inches in diameter, into seven equal divisions.

Table of Fractions of π

All hand-books contain tables giving multiples of π, but it often happens that the value of π divided by a given whole number is required. The table on page 20 has been calculated for this purpose, giving the quotient of π divided by all whole numbers from 1 to 100. From this table, for example, let us find the value of $\pi \div 38$. The divisor, 38, is located in the column headed a, alongside of which, under $\frac{\pi}{a}$, we find the quotient to be 0.08267.

Table of Inscribed Tangent Circles

On page 21 a table is given of the number of inscribed circles of a certain diameter that can be placed inside of a larger circle of a given diameter. In the table, the columns headed "N" give the number of inscribed circles, and the columns headed "R," the ratio of the diameter of the large circle to the diameter of the small circle. The values N and R, as given in the table, were found by the approximate formulas given at the head of the table, the large and small diameters being given. The example at the top illustrates its use.

Angles of Rise in Twelve Inches by Thirty-seconds

On pages 22 to 24 are given tables of angles of rise in twelve inches, by thirty-seconds, and on page 25 an example is shown of the application of these tables to practical problems. When the angle in degrees and minutes or in total minutes is known, the corresponding rise in 12 inches is found in the two left-hand columns headed "Inches" and "32nds," respectively. The third column, headed "Decimals," gives the same length as in the two first columns, but in decimals of a foot. As explained in the note on page 25, the columns of total minutes were included in order to permit the use of the total number of minutes directly, as this is more convenient for multiplication and division, and even for addition and subtraction, and the chances for error are reduced.

In the example given on page 25 the angles are first found by ordinary trigonometrical calculation, and when the angles have been found they are located in the tables and the corresponding rise determined without calculation. [MACHINERY'S Reference Series No. 52, Advanced Shop Arithmetic for the Machinist, Chapter XI, Use of Tables of Sines, Cosines, Tangents and Cotangents; MACHINERY'S Reference Series No. 54, The Solution of Triangles.]

Lengths of Chords for Spacing Circle whose Diameter is 1.— For Circles of other Diameters Multiply Length Given in Table by Diameter of Circle.

No. of Spaces.	Length of Chord.	No. of Spaces.	Length of Chord.	No. of Spaces.	Length of Chord.	No. of Spaces.	Length of Chord.
		26	.1205	51	.0616	76	.0413
		27	.1161	52	.0604	77	.0408
3	.8660	28	.1120	53	.0592	78	.0403
4	.7071	29	.1081	54	.0581	79	.0398
5	.5878	30	.1045	55	.0571	80	.0393
6	.5000	31	.1012	56	.0561	81	.0388
7	.4339	32	.0980	57	.0551	82	.0383
8	.3827	33	.0951	58	.0541	83	.0378
9	.3420	34	.0923	59	.0532	84	.0374
10	.3090	35	.0896	60	.0523	85	.0370
11	.2817	36	.0872	61	.0515	86	.0365
12	.2588	37	.0848	62	.0507	87	.0361
13	.2393	38	.0826	63	.0499	88	.0357
14	.2225	39	.0805	64	.0491	89	.0353
15	.2079	40	.0785	65	.0483	90	.0349
16	.1951	41	.0765	66	.0476	91	.0345
17	.1838	42	.0747	67	.0469	92	.0341
18	.1736	43	.0730	68	.0462	93	.0338
19	.1646	44	.0713	69	.0455	94	.0334
20	.1564	45	.0698	70	.0449	95	.0331
21	.1490	46	.0682	71	.0442	96	.0327
22	.1423	47	.0668	72	.0436	97	.0324
23	.1362	48	.0654	73	.0430	98	.0321
24	.1305	49	.0641	74	.0424	99	.0317
25	.1253	50	.0628	75	.0419	100	.0314

Right-angled Triangles

In right-angled triangles the remaining sides and angles can be found when either two sides, or one side and one of the acute angles, are known. The side opposite the right angle, or side a in Fig. 1, is called the *hypotenuse*. Side b is the side adjacent to angle C and side c is the side opposite the same angle. In the same way, c is the side *adjacent* to angle B, and b is the side *opposite* angle B.

The problems in right-angled triangles may be divided into five classes, for

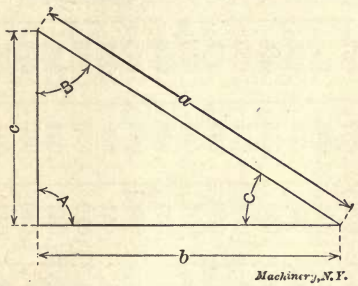


Fig. 1. Right-angled Triangle

which the following formulas are given:

1. The hypotenuse and one of the sides forming the right angle are given. Call the hypotenuse a, and the other given side b. Then (see Fig. 1):

$$c = \sqrt{a^2 - b^2} \qquad \sin B = \frac{b}{a}$$

$$C = 90° - B$$

2. The two sides forming the right angle are given. Call these two sides b and c. Then (see Fig. 1):

$$a = \sqrt{b^2 + c^2} \qquad \tan B = \frac{b}{c}$$

$$C = 90° - B$$

3. The hypotenuse and one of the acute angles are given. Call the hypotenuse a and the given angle B. Then (see Fig. 1):

$$C = 90° - B \qquad b = a \times \sin B$$

$$c = a \times \cos B$$

4. One acute angle and its adjacent side are given. Call the given angle B and its adjacent side c. Then (see Fig. 1):

$$C = 90° - B \qquad a = \frac{c}{\cos B}$$

$$b = c \times \tan B$$

5. One acute angle and its opposite side are given. Call the given angle B and the side opposite it b. Then (see Fig. 1):

$$C = 90° - B \qquad a = \frac{b}{\sin B}$$

$$c = b \times \cot B$$

On page 26 a diagrammatical arrangement for aid in the solution of right-angled triangles is given. Here each of the various cases has been reduced to its simplest form and the formulas given in as simple a manner as possible, so as to make the table especially convenient for the use of the beginner, and the person unfamiliar with the solution of triangles. On page 27 another table for the solving of right-angled triangles, arranged in a somewhat different manner, is given. It is understood that the formulas just given in the explanatory matter above, and the formulas on pages 26 and 27 all give the same results, but the different arrangements have been made only in order to suit the requirements of different users. Those who prefer formulas in their concrete form will use those given in the explanatory matter, whereas those who prefer to see each case clearly outlined, will find the tables on pages 26 and 27 more convenient. [MACHINERY'S Reference Series No. 54, The Solution of Triangles.]

Formulas for Solving Oblique Triangles

Following is a summary of all the generally required formulas, and the methods of procedure in solving oblique triangles. In all the formulas reference is made to Fig. 2, in which the sides

(Continued on page 29.)

TABLE FOR SPACING OFF CIRCLES.

No. of Divisions.	Degrees in Arc.	Length of chord. Diam. 1/16.	1/8	3/16	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8	15/16
3	120	.054	.108	.162	.217	.270	.324	.378	.433	.486	.540	.594	.651	.702	.756	.810
4	90	.044	.088	.132	.176	.220	.264	.308	.354	.396	.440	.484	.528	.572	.616	.660
5	72	.037	.073	.111	.147	.185	.219	.259	.294	.333	.365	.407	.441	.481	.514	.555
6	60	.031	.063	.093	.125	.155	.188	.217	.250	.279	.315	.341	.375	.403	.438	.465
7	51 3/7	.027	.054	.081	.108	.135	.162	.189	.217	.244	.271	.298	.325	.352	.379	.406
8	45	.024	.048	.072	.096	.120	.143	.167	.191	.215	.240	.263	.287	.310	.334	.358
9	40	.021	.043	.064	.086	.107	.128	.149	.171	.192	.214	.235	.257	.278	.297	.320
10	36	.019	.039	.058	.077	.097	.116	.135	.155	.174	.193	.212	.232	.251	.270	.289
11	32 8/11	.019	.035	.053	.070	.088	.105	.123	.141	.158	.175	.193	.211	.228	.246	.263
12	30	.016	.032	.048	.065	.081	.097	.113	.129	.145	.160	.177	.194	.209	.223	.242
13	27 9/13	.015	.029	.045	.059	.075	.089	.105	.119	.134	.149	.164	.178	.194	.209	.224
14	25 5/7	.014	.028	.042	.056	.069	.083	.097	.111	.125	.139	.153	.167	.180	.194	.208
15	24	.013	.026	.039	.052	.065	.078	.091	.104	.116	.129	.142	.155	.168	.181	.194
16	22 1/2	.012	.024	.037	.049	.060	.073	.085	.098	.109	.122	.134	.146	.158	.170	.183
17	21 8/17	.011	.023	.034	.046	.057	.069	.080	.092	.103	.114	.126	.138	.149	.160	.172
18	20	.011	.022	.032	.043	.054	.065	.076	.087	.097	.108	.119	.129	.141	.151	.162
19	18 18/19	.010	.021	.031	.041	.051	.062	.071	.082	.092	.103	.113	.123	.133	.144	.154
20	18	.010	.020	.029	.039	.049	.059	.068	.078	.088	.098	.107	.117	.127	.136	.146
21	17 1/7	.009	.019	.028	.037	.047	.056	.065	.075	.084	.094	.102	.112	.121	.130	.139
22	16 4/11	.009	.018	.027	.036	.045	.053	.062	.071	.080	.089	.098	.107	.115	.124	.133
23	15 15/23	.009	.017	.026	.034	.043	.051	.059	.068	.077	.085	.094	.102	.111	.118	.128
24	15	.008	.016	.024	.033	.041	.049	.057	.065	.073	.081	.089	.098	.107	.114	.122
25	14 2/5	.008	.016	.023	.031	.039	.047	.055	.063	.070	.078	.086	.094	.102	.109	.117
26	13 11/13	.008	.015	.023	.030	.038	.045	.053	.060	.068	.075	.083	.090	.098	.105	.113
27	13 1/3	.007	.015	.022	.029	.036	.044	.051	.058	.065	.073	.080	.087	.094	.102	.109
28	12 6/7	.007	.014	.021	.028	.035	.042	.049	.056	.063	.070	.077	.084	.091	.098	.105
29	12 12/29	.007	.014	.020	.027	.034	.041	.047	.054	.061	.068	.074	.081	.088	.095	.101
30	12	.007	.013	.019	.026	.033	.039	.046	.052	.059	.065	.072	.078	.085	.091	.098
31	11 11/31	.006	.013	.019	.025	.032	.038	.044	.051	.057	.063	.069	.076	.082	.088	.095
32	11 1/4	.006	.012	.018	.024	.031	.037	.043	.049	.055	.061	.067	.074	.079	.086	.092
33	10 10/11	.006	.012	.018	.023	.029	.036	.042	.048	.054	.059	.065	.071	.077	.083	.089

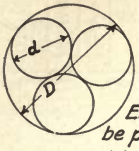
TABLE FOR SPACING OFF CIRCLES.

No. of Divisions.	Degrees in Arc.	Length of chord. Diam. 1.	2	3	4	5	6	7	8	9	10	11	12	13	14
3	120	.866	1.732	2.598	3.464	4.330	5.196	6.062	6.928	7.794	8.666	9.526	10.392	11.258	12.124
4	90	.707	1.414	2.121	2.828	3.536	4.243	4.950	5.657	6.364	7.081	7.778	8.485	9.192	9.900
5	72	.588	1.176	1.763	2.351	2.938	3.527	4.115	4.702	5.290	5.878	6.465	7.053	7.641	8.229
6	60	.500	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	5.000	5.500	6.000	6.500	7.000
7	51 $\frac{3}{7}$	.434	.868	1.302	1.736	2.170	2.604	3.037	3.471	3.905	4.339	4.773	5.207	5.641	6.075
8	45	.383	.765	1.148	1.531	1.913	2.296	2.679	3.061	3.444	3.827	4.210	4.592	4.975	5.358
9	40	.342	.684	1.026	1.368	1.710	2.052	2.394	2.736	3.078	3.420	3.762	4.104	4.446	4.788
10	36	.309	.618	.927	1.236	1.545	1.854	2.163	2.472	2.781	3.090	3.399	3.708	4.017	4.326
11	32 $\frac{8}{11}$	.282	.564	.845	1.127	1.409	1.691	1.973	2.254	2.536	2.818	3.100	3.381	3.663	3.946
12	30	.259	.518	.776	1.035	1.294	1.553	1.812	2.069	2.329	2.588	2.847	3.106	3.365	3.624
13	27 $\frac{9}{13}$	.239	.479	.718	.958	1.197	1.436	1.676	1.915	2.154	2.394	2.633	2.873	3.112	3.352
14	25 $\frac{5}{7}$	.222	.445	.667	.890	1.112	1.334	1.557	1.779	2.000	2.224	2.446	2.669	2.891	3.114
15	24	.208	.416	.624	.832	1.040	1.247	1.455	1.663	1.871	2.079	2.287	2.495	2.703	2.901
16	22 $\frac{1}{2}$	.195	.390	.585	.780	.975	1.171	1.366	1.561	1.756	1.951	2.146	2.341	2.536	2.731
17	21 $\frac{2}{17}$	.184	.367	.551	.735	.918	1.102	1.286	1.469	1.653	1.837	2.020	2.204	2.388	2.571
18	20	.174	.347	.521	.695	.868	1.041	1.216	1.389	1.563	1.737	1.910	2.084	2.257	2.431
19	18 $\frac{18}{19}$	.164	.329	.493	.658	.822	.987	1.151	1.316	1.480	1.645	1.809	1·974	2.137	2.303
20	18	.156	.318	.469	.626	.782	.937	1.095	1.251	1.408	1.564	1.721	1.877	2.033	2.190
21	17 $\frac{1}{7}$	.149	.298	.447	.596	.745	.894	1.043	1.192	1.341	1.489	1.639	1.788	1.936	2.085
22	16 $\frac{4}{11}$	.142	.286	.427	.569	.712	.855	.996	1.139	1.281	1.423	1.566	1.708	1.850	1.993
23	15 $\frac{15}{23}$	.136	.273	.409	.545	.681	.818	.954	1.092	1.227	1.363	1.499	1.635	1.772	1.908
24	15	.130	.261	.392	.522	.653	.783	.914	1.044	1.175	1.305	1.436	1.566	1.697	1.827
25	14 $\frac{2}{5}$	.125	.251	.375	.501	.627	.752	.877	1.003	1.128	1.253	1.379	1.504	1.629	1.755
26	13 $\frac{11}{13}$	.120	.241	.361	.482	.602	.723	.843	.963	1.084	1.204	1.325	1.445	1.566	1.686
27	13 $\frac{1}{3}$	.116	.232	.348	.464	.580	.697	.813	.929	1.045	1.161	1.277	1.393	1.509	1.625
28	12 $\frac{6}{7}$	.112	.224	.336	.448	.560	.672	.784	.896	1.008	1.121	1.233	1.345	1.457	1.569
29	12 $\frac{18}{29}$	.108	.216	.324	.432	.540	.648	.756	.864	.972	1.080	1.188	1.296	1.404	1.512
30	12	.104	.209	.314	.418	.522	.627	.732	.836	.941	1.045	1.150	1.254	1.359	1.463
31	11 $\frac{19}{31}$	.101	.202	.303	.404	.505	.606	.707	.808	.910	1.011	1.112	1.213	1.314	1.415
32	11 $\frac{1}{4}$	.098	.196	.294	.393	.491	.589	.687	.785	.883	.982	1.080	1.178	1.276	1.374
33	10 $\frac{10}{11}$	.095	.190	.285	.380	.475	.570	.665	.760	.855	.949	1.045	1.140	1.235	1.330

TABLE OF FRACTIONS OF $\pi = 3.14159265$.

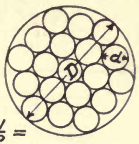
a	$\frac{\pi}{a}$	a	$\frac{\pi}{a}$	a	$\frac{\pi}{a}$	a	$\frac{\pi}{a}$	a	$\frac{\pi}{a}$	a	$\frac{\pi}{a}$
1	3.14159	18	.17453	35	.08976	52	.06042	69	.04553	86	.03653
2	1.57080	19	.16535	36	.08727	53	.05928	70	.04488	87	.03611
3	1.04720	20	.15708	37	.08491	54	.05818	71	.04425	88	.03570
4	.78540	21	.14960	38	.08267	55	.05712	72	.04363	89	.03530
5	.62832	22	.14280	39	.08055	56	.05610	73	.04304	90	.03491
6	.52360	23	.13659	40	.07854	57	.05512	74	.04245	91	.03452
7	.44880	24	.13090	41	.07662	58	.05417	75	.04189	92	.03415
8	.39270	25	.12566	42	.07480	59	.05325	76	.04134	93	.03378
9	.34907	26	.12083	43	.07306	60	.05236	77	.04080	94	.03342
10	.31416	27	.11636	44	.07140	61	.05150	78	.04028	95	.03307
11	.28560	28	.11220	45	.06981	62	.05067	79	.03977	96	.03272
12	.26180	29	.10833	46	.06830	63	.04987	80	.03927	97	.03239
13	.24166	30	.10472	47	.06684	64	.04909	81	.03879	98	.03206
14	.22440	31	.10134	48	.06545	65	.04833	82	.03831	99	.03173
15	.20944	32	.09817	49	.06411	66	.04760	83	.03785	100	.03142
16	.19635	33	.09520	50	.06283	67	.04689	84	.03740	..	.....
17	.18480	34	.09240	51	.06160	68	.04620	85	.03696	...	.....

TABLE OF INSCRIBED TANGENT CIRCLES



Approximate Formulas:

$$N = 0.907\left(\frac{D}{d} - 0.94\right)^2 + 3.7. \qquad R = 0.94 + \sqrt{\frac{N - 3.7}{0.907}}.$$



N = Number of inscribed circles.
R = Diameter of large circle ÷ diameter of small circle.
Example:- How many wires $\frac{1}{2}$ inch in diameter can be placed inside a 5 inch pipe? The value of R is $5 \div \frac{1}{2} =$ 10, and looking for this number in table, find by interpolation N = 76.

N	R	N	R	N	R	N	R	N	R
2	2.00	34	6.76	130	12.80	290	18.75	600	26.65
3	2.15	35	6.86	135	13.06	295	18.90	610	26.86
4	2.41	36	7.00	140	13.26	300	19.05	620	27.07
5	2.70	37	7.00	145	13.49	310	19.35	630	27.28
6	3.00	38	7.08	150	13.72	320	19.65	640	27.49
7	3.00	39	7.18	155	13.95	330	19.94	650	27.70
8	3.31	40	7.31	160	14.17	340	20.23	660	27.91
9	3.61	41	7.39	165	14.39	350	20.52	670	28.12
10	3.80	42	7.43	170	14.60	360	20.81	680	28.33
11	3.92	43	7.61	175	14.81	370	21.09	690	28.54
12	4.05	44	7.70	180	15.01	380	21.36	700	28.75
13	4.23	45	7.72	185	15.20	390	21.63	720	29.14
14	4.41	46	7.81	190	15.39	400	21.90	740	29.52
15	4.55	47	7.92	195	15.57	410	22.17	760	29.90
16	4.70	48	8.00	200	15.75	420	22.44	780	30.28
17	4.86	49	8.03	205	15.93	430	22.70	800	30.65
18	5.00	50	8.13	210	16.11	440	22.96	820	31.02
19	5.00	55	8.21	215	16.29	450	23.21	840	31.39
20	5.18	60	8.94	220	16.46	460	23.47	860	31.75
21	5.31	65	9.25	225	16.63	470	23.72	880	32.11
22	5.49	70	9.61	230	16.80	480	23.97	900	32.46
23	5.61	75	9.93	235	16.97	490	24.21	920	32.80
24	5.72	80	10.20	240	17.14	500	24.45	940	33.14
25	5.81	85	10.46	245	17.30	510	24.68	960	33.48
26	5.92	90	10.73	250	17.46	520	24.91	980	33.82
27	6.00	95	11.15	255	17.63	530	25.13	1000	34.15
28	6.13	100	11.34	260	17.79	540	25.35	1100	35.75
29	6.23	105	11.60	265	17.95	550	25.57	1200	37.30
30	6.40	110	11.85	270	18.11	560	25.79	1300	38.80
31	6.44	115	12.10	275	18.27	570	26.01	1400	40.20
32	6.55	120	12.34	280	18.43	580	26.23	1500	41.60
33	6.70	125	12.57	285	18.59	590	26.44	1600	42.95

ANGLES OF RISE IN 12 INCHES, BY 32nds—I

Inches	32nds.	Decimals	Degrees	Minutes	Total Minutes	Inches	32nds.	Decimals	Degrees	Minutes	Total Minutes	Inches	32nds.	Decimals	Degrees	Minutes	Total Minutes
0	0					1	11	0.1120		23	383	2	22	0.2240		37	757
	1	0.0026	0	9	9		12	0.1146		32	392		23	0.2266		46	766
	2	0.0052		18	18		13	0.1172		41	401		24	0.2292		54	774
	3	0.0078		27	27		14	0.1198		50	410		25	0.2318	13	3	783
	4	0.0104		36	36		15	0.1224		59	419		26	0.2344		11	791
	5	0.0130		45	45		16	0.1250	7	8	428		27	0.2370		20	800
	6	0.0156		54	54		17	0.1276		16	436		28	0.2396		28	808
	7	0.0182	1	3	63		18	0.1302		25	445		29	0.2422		37	817
	8	0.0208		12	72		19	0.1328		34	454		30	0.2448		45	825
	9	0.0234		21	81		20	0.1354		43	463		31	0.2474		54	834
	10	0.0260		30	90		21	0.1380		51	471	3	0	0.2500	14	2	842
	11	0.0286		38	98		22	0.1406	8	0	480		1	0.2526		11	851
	12	0.0313		47	107		23	0.1432		9	489		2	0.2552		19	859
	13	0.0339		56	116		24	0.1458		18	498		3	0.2578		27	867
	14	0.0365	2	5	125		25	0.1484		27	507		4	0.2604		36	876
	15	0.0391		14	134		26	0.1510		35	515		5	0.2630		44	884
	16	0.0417		23	143		27	0.1536		44	524		6	0.2656		52	892
	17	0.0443		32	152		28	0.1563		53	533		7	0.2682	15	1	901
	18	0.0469		41	161		29	0.1589	9	2	542		8	0.2708		9	909
	19	0.0495		50	170		30	0.1615		10	550		9	0.2734		18	918
	20	0.0521		59	179		31	0.1641		19	559		10	0.2760		26	926
	21	0.0547	3	8	188	2	0	0.1667		28	568		11	0.2786		34	934
	22	0.0573		17	197		1	0.1693		36	576		12	0.2813		43	943
	23	0.0599		26	206		2	0.1719		45	585		13	0.2839		51	951
	24	0.0625		35	215		3	0.1745		54	594		14	0.2865		59	959
	25	0.0651		44	224		4	0.1771	10	3	603		15	0.2891	16	7	967
	26	0.0677		53	233		5	0.1797		11	611		16	0.2917		16	976
	27	0.0703	4	1	241		6	0.1823		20	620		17	0.2943		24	984
	28	0.0729		10	250		7	0.1849		29	629		18	0.2969		32	992
	29	0.0755		19	259		8	0.1875		37	637		19	0.2995		40	1000
	30	0.0781		28	268		9	0.1901		46	646		20	0.3021		48	1008
	31	0.0807		37	277		10	0.1927		54	654		21	0.3047		57	1017
1	0	0.0833		46	286		11	0.1953	11	3	663		22	0.3073	17	5	1025
	1	0.0859		55	295		12	0.1979		12	672		23	0.3099		13	1033
	2	0.0885	5	4	304		13	0.2005		20	680		24	0.3125		21	1041
	3	0.0911		12	312		14	0.2031		29	689		25	0.3151		29	1049
	4	0.0938		21	321		15	0.2057		37	697		26	0.3177		37	1057
	5	0.0964		30	330		16	0.2083		46	706		27	0.3203		46	1066
	6	0.0990		39	339		17	0.2109		55	715		28	0.3229		54	1074
	7	0.1016		48	348		18	0.2135	12	3	723		29	0.3255	18	2	1082
	8	0.1042		57	357		19	0.2161		12	732		30	0.3281		10	1090
	9	0.1068	6	6	366		20	0.2188		20	740		31	0.3307		18	1098
	10	0.1094		15	375		21	0.2214		29	749						

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ANGLES OF RISE IN 12 INCHES, BY 32nds—II

Inches	32nds.	Decimals	Degrees	Minutes	Total Minutes	Inches	32nds.	Decimals	Degrees	Minutes	Total Minutes	Inches	32nds.	Decimals	Degrees	Minutes	Total Minutes
4	0	0.3333	18	26	1106	5	11	0.4453	24	0	1440	6	22	0.5573		8	1748
	1	0.3359		34	1114		12	0.4479		8	1448		23	0.5599		15	1755
	2	0.3385		42	1122		13	0.4505		15	1455		24	0.5625		21	1761
	3	0.3411		50	1130		14	0.4531		23	1463		25	0.5651		28	1768
	4	0.3438		58	1138		15	0.4557		30	1470		26	0.5677		35	1775
	5	0.3464	19	6	1146		16	0.4583		37	1477		27	0.5703		42	1782
	6	0.3490		14	1154		17	0.4609		45	1485		28	0.5729		49	1787
	7	0.3516		22	1162		18	0.4635		52	1492		29	0.5755		55	1795
	8	0.3542		30	1170		19	0.4661		59	1499		30	0.5781	30	2	1802
	9	0.3568		38	1178		20	0.4688	25	7	1507		31	0.5807		9	1809
	10	0.3594		46	1186		21	0.4714		14	1514	7	0	0.5833		15	1815
	11	0.3620		54	1194		22	0.4740		22	1522		1	0.5859		22	1822
	12	0.3646	20	2	1202		23	0.4766		29	1529		2	0.5885		29	1829
	13	0.3672		10	1210		24	0.4792		36	1536		3	0.5911		35	1835
	14	0.3698		18	1218		25	0.4818		43	1543		4	0.5937		42	1842
	15	0.3724		26	1226		26	0.4844		51	1551		5	0.5964		49	1849
	16	0.3750		33	1233		27	0.4870		58	1558		6	0.5990		55	1855
	17	0.3776		41	1241		28	0.4896	26	5	1565		7	0.6016	31	2	1862
	18	0.3802		49	1249		29	0.4922		12	1572		8	0.6042		8	1868
	19	0.3828		57	1257		30	0.4949		20	1580		9	0.6068		15	1875
	20	0.3854	21	5	1265		31	0.4974		27	1587		10	0.6094		21	1881
	21	0.3880		12	1272	6	0	0.5000		34	1594		11	0.6120		28	1888
	22	0.3906		20	1280		1	0.5026		41	1601		12	0.6146		35	1895
	23	0.3932		28	1288		2	0.5052		48	1608		13	0.6172		41	1901
	24	0.3958		36	1296		3	0.5078		55	1615		14	0.6198		47	1907
	25	0.3984		43	1303		4	0.5104	27	2	1622		15	0.6224		54	1914
	26	0.4010		51	1311		5	0.5130		10	1630		16	0.6250	32	0	1920
	27	0.4036		59	1319		6	0.5156		17	1637		17	0.6276		7	1927
	28	0.4063	22	7	1327		7	0.5182		24	1644		18	0.6302		13	1933
	29	0.4089		14	1334		8	0.5208		31	1651		19	0.6328		20	1940
	30	0.4115		22	1342		9	0.5234		38	1658		20	0.6354		26	1946
	31	0.4141		30	1350		10	0.5260		45	1665		21	0.6380		32	1952
5	0	0.4167		37	1357		11	0.5286		52	1672		22	0.6406		39	1959
	1	0.4193		45	1365		12	0.5312		59	1679		23	0.6432		45	1965
	2	0.4219		52	1372		13	0.5339	28	6	1686		24	0.6458		51	1971
	3	0.4245	23	0	1380		14	0.5365		13	1693		25	0.6484		58	1978
	4	0.4271		8	1388		15	0.5391		20	1700		26	0.6510	33	4	1984
	5	0.4297		15	1395		16	0.5417		27	1707		27	0.6536		10	1990
	6	0.4323		23	1403		17	0.5443		34	1714		28	0.6562		16	1996
	7	0.4349		30	1410		18	0.5469		40	1720		29	0.6589		23	2003
	8	0.4375		38	1418		19	0.5495		47	1727		30	0.6615		29	2009
	9	0.4401		45	1425		20	0.5521		54	1734		31	0.6641		35	2015
	10	0.4427		53	1433		21	0.5547	29	1	1741						

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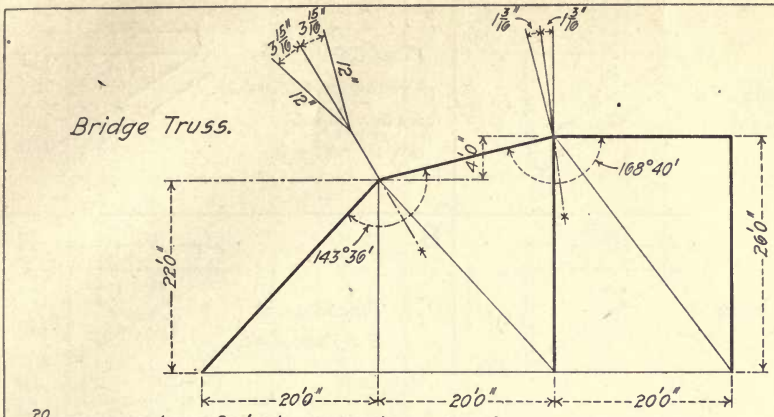
ANGLES OF RISE IN 12 INCHES, BY 32nds—III

Inches	32nds.	Decimals	Degrees	Minutes	Total Minutes	Inches	32nds.	Decimals	Degrees	Minutes	Total Minutes	Inches	32nds.	Decimals	Degrees	Minutes	Total Minutes
8	0	0.6667	33	41	2021	9	11	0.7786		54	2274		22	0.8906	41	41	2501
	1	0.6693		48	2028		12	0.7812	38	0	2280		23	0.8932		46	2506
	2	0.6719		54	2034		13	0.7839		6	2286		24	0.8958		51	2511
	3	0.6745	34	0	2040		14	0.7865		11	2291		25	0.8984		56	2516
	4	0.6771		6	2046		15	0.7891		17	2297		26	0.9010	42	1	2521
	5	0.6797		12	2052		16	0.7917		22	2302		27	0.9036		6	2526
	6	0.6823		18	2058		17	0.7943		28	2308		28	0.9062		11	2531
	7	0.6849		24	2064		18	0.7969		33	2313		29	0.9089		16	2536
	8	0.6875		31	2071		19	0.7995		39	2319		30	0.9115		21	2541
	9	0.6901		37	2077		20	0.8021		44	2324		31	0.9141		26	2546
	10	0.6927		43	2083		21	0.8047		49	2329	11	0	0.9167		31	2551
	11	0.6953		49	2089		22	0.8073		55	2335		1	0.9193		36	2556
	12	0.6979		55	2095		23	0.8099	39	0	2340		2	0.9219		40	2560
	13	0.7005	35	1	2101		24	0.8125		6	2346		3	0.9245		45	2565
	14	0.7031		7	2107		25	0.8151		11	2351		4	0.9271		50	2570
	15	0.7057		13	2113		26	0.8177		16	2356		5	0.9297		55	2575
	16	0.7083		19	2119		27	0.8203		22	2362		6	0.9323	43	0	2580
	17	0.7109		25	2125		28	0.8229		27	2367		7	0.9349		4	2584
	18	0.7135		31	2131		29	0.8255		32	2372		8	0.9375		9	2589
	19	0.7161		37	2137		30	0.8281		38	2378		9	0.9401		14	2594
	20	0.7188		43	2143		31	0.8307		43	2383		10	0.9427		19	2599
	21	0.7214		48	2148	10	0	0.8333		48	2388		11	0.9453		23	2603
	22	0.7240		54	2154		1	0.8359		54	2394		12	0.9479		28	2608
	23	0.7266	36	0	2160		2	0.8385		59	2399		13	0.9505		33	2613
	24	0.7292		6	2166		3	0.8411	40	4	2404		14	0.9531		38	2618
	25	0.7318		12	2172		4	0.8437		9	2409		15	0.9557		42	2622
	26	0.7344		18	2178		5	0.8464		15	2415		16	0.9583		47	2627
	27	0.7370		24	2184		6	0.8490		20	2420		17	0.9609		52	2632
	28	0.7396		29	2189		7	0.8516		25	2425		18	0.9635		56	2636
	29	0.7422		35	2195		8	0.8542		30	2430		19	0.9661	44	1	2641
	30	0.7448		41	2201		9	0.8568		35	2435		20	0.9688		6	2646
	31	0.7474		46	2206		10	0.8594		41	2441		21	0.9714		10	2650
9	0	0.7500		52	2212		11	0.8620		46	2446		22	0.9740		15	2655
	1	0.7526		58	2218		12	0.8646		51	2451		23	0.9766		19	2659
	2	0.7552	37	4	2224		13	0.8672		56	2456		24	0.9792		24	2664
	3	0.7578		9	2229		14	0.8698	41	1	2461		25	0.9818		28	2668
	4	0.7604		15	2235		15	0.8724		6	2466		26	0.9844		33	2673
	5	0.7630		21	2241		16	0.8750		11	2471		27	0.9870		38	2678
	6	0.7656		26	2246		17	0.8776		16	2476		28	0.9896		42	2682
	7	0.7682		32	2252		18	0.8802		21	2481		29	0.9922		47	2687
	8	0.7708		38	2258		19	0.8828		26	2486		30	0.9948		51	2691
	9	0.7734		43	2263		20	0.8854		31	2491		31	0.9974	44	56	2696
	10	0.7760		49	2269		21	0.8880		36	2496						

ANGLES OF RISE IN 12 INCHES, BY 32nds—IV



$\frac{20}{22}$ = 0.9090 = tan 42°16′ = tan 2536 Min.

$\frac{4}{20}$ = 0.2000 = tan 11°20′ = tan 680 Min.

90°0′ = 5400 Min.

8616 Min.

$\frac{8616}{2}$ = 4308 Min.; 5400 − 4308 = 1092 Min.;

1092 Min. = 18°10′, or $3\frac{15}{16}$" in 12".

$\frac{4}{20}$ = 0.200 = tan 11°20′ = tan 680 Min.

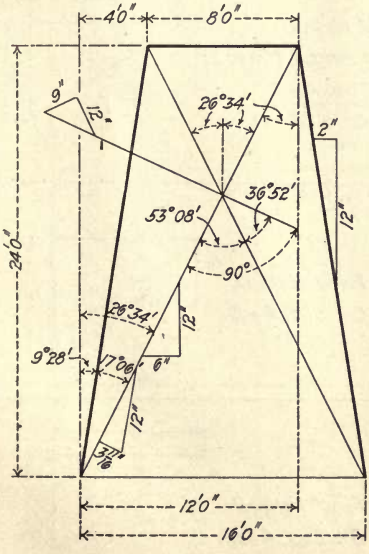
90° = 5400 Min.; 5400 − 680 = 4720 Min.;

4720 + 5400 = 10120 Min.;

$\frac{10120}{2}$ = 5060; 5400 − 5060 = 340 Min.;

340 Min. = 5°40′, or $1\frac{3}{16}$" in 12".

45° = 2700 Min. 90° = 5400 Min. 180° = 10800 Min. 360° = 21600 Min.



Trestle Bend
1/6" Batter.

$\frac{12}{24}$ = 0.5 = tan 26°34′, or 6" in 12"

$\frac{4}{24}$ = 0.1666 = tan 9°28′ = 568 Min.;

26°34′ = 1594 Min.; 1594 − 568 = 1026 Min., or $3\frac{11}{16}$" in 12".

26°34′ × 2 = 53°8′ = 3188 Min.

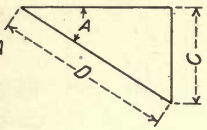
5400 − 3188 = 2212 Min. = 36°52′, or 9" in 12".

Note: The above examples are given to explain the use of the tables. For general convenience the tables were made to include columns of total minutes as well as columns of degrees and minutes, thus permitting the use of total minutes directly from the tables if preferred instead of degrees and minutes. The use of total minutes is more convenient for multiplication and division or even for addition and subtraction, and the chances for errors are reduced.

SOLUTION OF RIGHT-ANGLED TRIANGLES

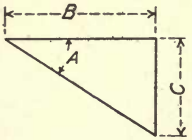
1

Find Angle A
C ÷ D = Sin. A



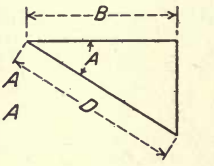
2

Find Angle A
C ÷ B = Tan. A



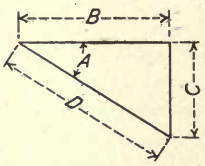
3

Find Angle A
B ÷ D = Cos. A



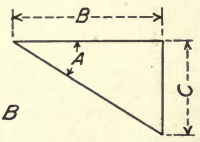
4

Find Adj. B
{Find Ang. A (Case 1)
{D x Cos. A = B
or $\sqrt{D^2 - C^2} = B$



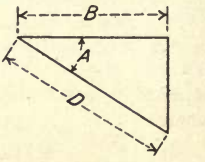
5

Find Adj. B
C x Cotan. A = B



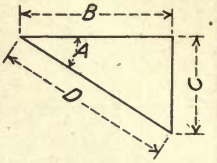
6

Find Adj. B
D x Cos. A = B



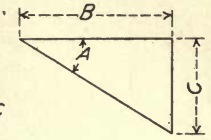
7

Find Opp. C
{Find Ang. A (Case 3)
{D x Sin. A = C
or $\sqrt{D^2 - B^2} = C$



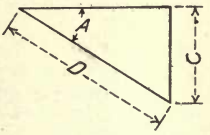
8

Find Opp. C
B x Tan. A = C



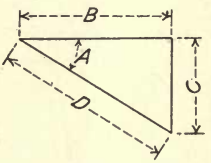
9

Find Opp. C
D x Sin. A = C



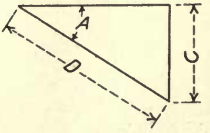
10

Find Hyp. D
{Find Ang. A (Case 2)
{B ÷ Cos. A = D
or $\sqrt{C^2 + B^2} = D$



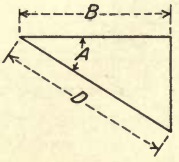
11

Find Hyp. D
C ÷ Sin. A = D



12

Find Hyp. D
B ÷ Cos. A = D



Contributed by F. J. Bouve. Explanatory note: Page 17.

TABLE FOR SOLVING RIGHT-ANGLED TRIANGLES.

PARTS GIVEN.	PARTS TO BE FOUND.				
	Hypotenuse (Hyp.)	Adjacent Side. (Adj.)	Opposite Side (Opp.)	Angle.	Opposite Angle.
Hyp. and Adj.	——	——	$\sqrt{\overline{Hyp.}^2 - \overline{Adj.}^2}$	$Cos. = \frac{Adj.}{Hyp.}$	$Sin. = \frac{Adj.}{Hyp.}$
Hyp. and Opp.	——	$\sqrt{\overline{Hyp.}^2 - \overline{Opp.}^2}$	——	$Sin. = \frac{Opp.}{Hyp.}$	$Cos. = \frac{Opp.}{Hyp.}$
Hyp. and Angle.	——	Hyp. × Cos.	Hyp. × Sin.	——	90° − Angle.
Adj. and Opp.	$\sqrt{\overline{Adj.}^2 + \overline{Opp.}^2}$	——	——	$Tang. = \frac{Opp.}{Adj.}$	$Cot. = \frac{Opp.}{Adj.}$
Adj. and Angle.	$\frac{Adj}{Cos.}$	——	Adj. × Tang.	——	90° − Angle.
Opp. and Angle.	$\frac{Opp.}{Sin.}$	Opp. × Cot.	——	——	90° − Angle

SOLUTION OF OBLIQUE-ANGLED TRIANGLES

Parts Given	Parts to be found.					
	$a=$	$b=$	$c=$	$\angle A$	$\angle B$	$\angle C$
$a-b-c$				$\frac{b^2+c^2-a^2}{2bc}=\cos A$	$\frac{a^2+c^2-b^2}{2ac}=\cos B$	$\frac{a^2+b^2-c^2}{2ab}=\cos C$
$b-c-\angle A$	$\sqrt{b^2+c^2-2bc\cos A}$				$\frac{b\sin A}{c-b\cos A}=\tan B$	$\frac{c\sin A}{b-c\cos A}=\tan C$
$a-c-\angle B$		$\sqrt{a^2+c^2-2ac\cos B}$		$\frac{a\sin B}{c-a\cos B}=\tan A$		$\frac{c\sin B}{a-c\cos B}=\tan C$
$a-b-\angle C$			$\sqrt{a^2+b^2-2ab\cos C}$	$\frac{a\sin C}{b-a\cos C}=\tan A$	$\frac{b\sin C}{a-b\cos C}=\tan B$	
$a-b-\angle A$			$\frac{a\sin C}{\sin A}$		$\frac{b\sin A}{a}=\sin B$	$180°-(A+B)$
$a-b-\angle B$			$\frac{b\sin C}{\sin B}$	$\frac{a\sin B}{b}=\sin A$		$180°-(A+B)$
$a-c-\angle A$		$\frac{a\sin B}{\sin A}$			$180°-(A+C)$	$\frac{c\sin A}{a}=\sin C$
$a-c-\angle C$		$\frac{c\sin B}{\sin C}$		$\frac{a\sin C}{c}=\sin A$	$180°-(A+C)$	
$b-c-\angle B$	$\frac{b\sin A}{\sin B}$			$180°-(B+C)$		$\frac{c\sin B}{b}=\sin C$
$b-c-\angle C$	$\frac{c\sin A}{\sin C}$			$180°-(B+C)$	$\frac{b\sin C}{c}=\sin B$	
$a-\angle A-\angle B$		$\frac{a\sin B}{\sin A}$	$\frac{a\sin C}{\sin A}$			$180°-(A+B)$
$a-\angle A-\angle C$		$\frac{a\sin B}{\sin A}$	$\frac{a\sin C}{\sin A}$		$180°-(A+C)$	
$a-\angle B-\angle C$		$\frac{a\sin B}{\sin A}$	$\frac{a\sin C}{\sin A}$	$180°-(B+C)$		
$b-\angle A-\angle B$	$\frac{b\sin A}{\sin B}$		$\frac{b\sin C}{\sin B}$			$180°-(A+B)$
$b-\angle A-\angle C$	$\frac{b\sin A}{\sin B}$		$\frac{b\sin C}{\sin B}$		$180°-(A+C)$	
$b-\angle B-\angle C$	$\frac{b\sin A}{\sin B}$		$\frac{b\sin C}{\sin B}$	$180°-(B+C)$		
$c-\angle A-\angle B$	$\frac{c\sin A}{\sin C}$	$\frac{c\sin B}{\sin C}$				$180°-(A+B)$
$c-\angle A-\angle C$	$\frac{c\sin A}{\sin C}$	$\frac{c\sin B}{\sin C}$			$180°-(A+C)$	
$c-\angle B-\angle C$	$\frac{c\sin A}{\sin C}$	$\frac{c\sin B}{\sin C}$		$180°-(B+C)$		

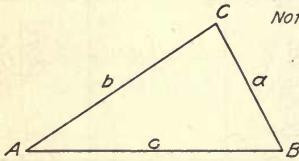


Fig. 1

Note 1:- By means of the table any part of an oblique triangle may be found when any three other parts are given, with the following exception:

Given two sides and the angle opposite one of them; then, if the side opposite is less than the adjacent × the sine of the angle, the triangle is impossible; or if the side opposite = the adjacent × the sine of the angle, the triangle is a right triangle; or if the side opposite is less than the adjacent but does not come under the above, the triangle is capable of two solutions and can be drawn as in Fig. 2 as well as in Fig. 1.

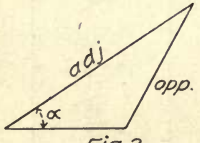


Fig. 2.

Note 2:- In some cases two steps are necessary to solve, as for example, having given sides a and b and angle A, to find c: The formula reads $c=\frac{a\sin C}{\sin A}$ but angle C must first be derived from $C=180°-(A+B)$, and the same applies to other angles in certain cases as is apparent above.

and angles are given the same names as in the formulas following.

1. When two angles and one side are given, call the given side a, the angle opposite it A, and the other angle B. Then if A is known:

$$C = 180° - (A + B) \qquad b = \frac{a \times \sin B}{\sin A}$$

$$c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

If B and C are given, but not A, then $A = 180° - (B + C)$, the other formulas being as above.

2. When two sides and the included angle are given, call the given sides a

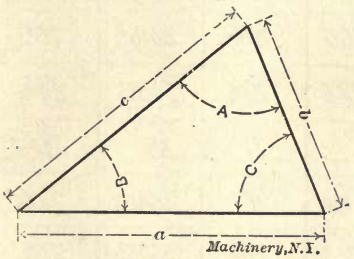


Fig. 2. Oblique-angled Triangle

and b and the given angle between them C. Then:

$$\tan A = \frac{a \times \sin C}{b - a \times \cos C}$$

$$B = 180° - (A + C)$$

$$c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

3. When two sides and the angle opposite one of the sides are given, call the given angle A, the side opposite it a and the other given side b. Then:

$$\sin B = \frac{b \times \sin A}{a}$$

$$C = 180° - (A + B)$$

$$c = \frac{a \times \sin C}{\sin A}$$

$$\text{Area} = \frac{a \times b \times \sin C}{2}$$

4. When the three sides of a triangle are given, call them a, b and c and the angles opposite them A, B and C re spectively. Then:

$$\cos A = \frac{b^2 + c^2 - a^2}{2 \times b \times c}$$

$$\sin B = \frac{b \times \sin A}{a}$$

$$C = 180° - (A + B)$$

$$\text{Area} = \frac{a \times b \times \cos C}{2}$$

The table on page 28 may be preferred by some to the formulas given above. By means of this table any part of an oblique triangle may be found when any three parts are given. The parts given are found in the left-hand column, where a, b, and c denote the sides, and $\angle A$, $\angle B$ and $\angle C$, angles A, B, and C respectively, the same as in the triangle Fig. 1 shown in the lower part on page 28. When the line with the parts given has been located in the left-hand column of the table, page 28, simply use the formulas in the same line for finding the sides or angles which are not known. In some cases two steps are necessary for the solution; for example, if the sides a and b and the angle A are given, to find the side c the formula is

$$c = \frac{a \sin C}{\sin A},$$

but angle B must first be found from the formula given for the sine for this angle, and then angle C is found from the formula

$$C = 180° - (A + B).$$

After these operations are completed the formula for c be applied. [MACHINERY'S Reference Series No. 54, The Solution of Triangles.]

DECIMALS OF A MINUTE FOR EACH QUARTER SECOND—I

Sec.	Dec.	Sec.	Dec.	Sec.	Dec.	Sec.	Dec.	Sec.	Dec.
1/4	.00416	6 1/4	.10416	12 1/4	.20416	18 1/4	.30416	24 1/4	.40416
1/2	.00833	6 1/2	.10833	12 1/2	.20833	18 1/2	.30833	24 1/2	.40833
3/4	.0125	6 3/4	.1125	12 3/4	.2125	18 3/4	.3125	24 3/4	.4125
1	.01666	7	.11666	13	.21666	19	.31666	25	.41666
1 1/4	.02083	7 1/4	.12083	13 1/4	.22083	19 1/4	.32083	25 1/4	.42083
1 1/2	.025	7 1/2	.125	13 1/2	.225	19 1/2	.325	25 1/2	.425
1 3/4	.02916	7 3/4	.12916	13 3/4	.22916	19 3/4	.32916	25 3/4	.42916
2	.03333	8	.13333	14	.23333	20	.33333	26	.43333
2 1/4	.0375	8 1/4	.1375	14 1/4	.2375	20 1/4	.3375	26 1/4	.4375
2 1/2	.04166	8 1/2	.14166	14 1/2	.24166	20 1/2	.34166	26 1/2	.44166
2 3/4	.04583	8 3/4	.14583	14 3/4	24583	20 3/4	.34583	26 3/4	.44583
3	.05	9	.15	15	.25	21	.35	27	.45
3 1/4	.05416	9 1/4	.15416	15 1/4	.25416	21 1/4	.35416	27 1/4	.45416
3 1/2	.05833	9 1/2	.15833	15 1/2	.25833	21 1/2	.35833	27 1/2	45833
3 3/4	.0625	9 3/4	.1625	15 3/4	.2625	21 3/4	.3625	27 3/4	.4625
4	.06666	10	.16666	16	.26666	22	.36666	28	46666
4 1/4	.07083	10 1/4	.17083	16 1/4	.27083	22 1/4	.37083	28 1/4	.47083
4 1/2	.075	10 1/2	.175	16 1/2	.275	22 1/2	.375	28 1/2	.475
4 3/4	.07916	10 3/4	.17916	16 3/4	.27916	22 3/4	.37916	28 3/4	.47916
5	.08333	11	.18333	17	.28333	23	38333	29	.48333
5 1/4	.0875	11 1/4	.1875	17 1/4	.2875	23 1/4	.3875	29 1/4	4875
5 1/2	.09166	11 1/2	.19166	17 1/2	.29166	23 1/2	.39166	29 1/2	.49166
5 3/4	.09583	11 3/4	.19583	17 3/4	.29583	23 3/4	.39583	29 3/4	.49583
6	.1	12	.2	18	.3	24	.4	30	.5

Contributed by H. S. Parks, MACHINERY'S Data Sheet No. 76. Explanatory note: Page 38.

DECIMALS OF A MINUTE FOR EACH QUARTER SECOND—II

Sec.	Dec.	Sec.	Dec.	Sec.	Dec.	Sec.	Dec.	Sec.	Dec.
30¼	.50416	36¼	.60416	42¼	70416	48¼	.80416	54¼	.90416
30½	.50833	36½	.60833	42½	.70833	48½	.80833	54½	.90833
30¾	.5125	36¾	.6125	42¾	.7125	48¾	.8125	54¾	.9125
31	.51666	37	.61666	43	.71666	49	.81666	55	.91666
31¼	.52083	37¼	.62083	43¼	.72083	49¼	.82083	55¼	.92083
31½	.525	37½	.625	43½	.725	49½	825	55½	.925
31¾	.52916	37¾	.62916	43¾	.72916	49¾	.82916	55¾	.92916
32	.53333	38	.63333	44	.73333	50	.83333	56	93333
32¼	.5375	38¼	.6375	44¼	7375	50¼	.8375	56¼	.9375
32½	.54166	38½	.64166	44½	.74166	50½	.84166	56½	.94166
32¾	.54583	38¾	.64583	44¾	74583	50¾	.84583	56¾	.94583
33	55	39	.65	45	.75	51	85	57	.95
33¼	.55416	39¼	.65416	45¼	75416	51¼	.85416	57¼	.95416
33½	.55833	39½	.65833	45½	75833	51½	.85833	57½	.95833
33¾	.5625	39¾	.6625	45¾	.7625	51¾	.8625	57¾	.9625
34	.56666	40	.66666	46	.76666	52	.86666	58	.96666
34¼	.57083	40¼	.67083	46¼	.77083	52¼	.87083	58¼	.97083
34½	.575	40½	.675	46½	.775	52½	.875	58½	.975
34¾	.57916	40¾	.67916	46¾	.77916	52¾	.87916	58¾	.97916
35	.58333	41	.68333	47	.78333	53	.88333	59	.98333
35¼	.5875	41¼	.6875	47¼	.7875	53¼	.8875	59¼	.9875
35½	.59166	41½	.69166	47½	.79166	53½	.89166	59½	.99166
35¾	.59583	41¾	.69583	47¾	.79583	53¾	.89583	59¾	.99583
36	.6	42	.7	48	.8	54	.9	60	1.0

Contributed by H. S. Parks, MACHINERY'S Data Sheet No. 76. Explanatory note: Page 38.

FORMULAS AND FACTORS FOR SOLVING REGULAR POLYGONS—I

Z = angle subtended at center by side

P = perimeter of polygon

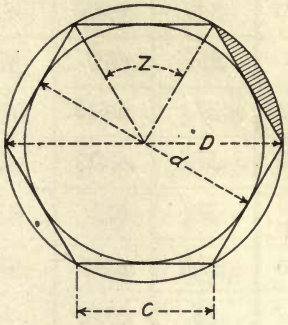
C = length of one side

A = area of polygon

N = number of sides

d = diameter of inscribed circle

D = diameter of circumscribed circle



In the formulas below, the trigonometrical expressions necessary to use for solving regular polygons are replaced by the letters B, F, H, K and M, the numerical values of which for the most common number of sides in polygons are given in table II. The expressions these letters signify are:

$B = \frac{1}{N} \operatorname{cosec} \frac{180^\circ}{N}$ $F = N \tan \frac{180^\circ}{N}$ $H = \frac{N}{8} \sin \frac{360^\circ}{N}$ $K = \frac{N}{4} \cot \frac{180^\circ}{N}$

$M = 2N \sin \frac{180^\circ}{N}$

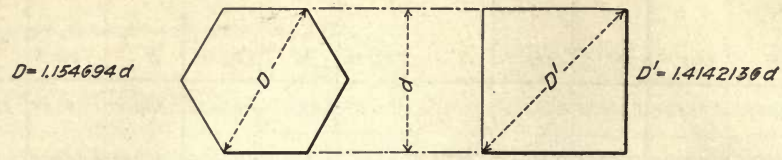
Quantities to be found.	Quantities Known.				
	P	A	C	D	d
P		$P = 2\sqrt{FA}$	$P = CN$	$P = \frac{MD}{2}$	$P = Fd$
A	$A = \frac{KP^2}{N^2}$		$A = KC^2$	$A = HD^2$	$A = \frac{Fd^2}{4}$
C	$C = \frac{P}{N}$	$C = \frac{2\sqrt{FA}}{N}$		$C = \frac{MD}{2N}$	$C = \frac{Fd}{N}$
D	$D = BP$	$D = 2B\sqrt{FA}$	$D = NBC$		$D = BFd$
d	$d = \frac{4KP}{N^2}$	$d = \frac{4\sqrt{AK}}{N}$	$d = \frac{4KC}{N}$	$d = \frac{2MKD}{N^2}$	

FORMULAS AND FACTORS FOR SOLVING REGULAR POLYGONS—II

Number of Sides N	F	Log. F	M	Log. M	H	Log. H	K	Log. K	B	Log. B	Center Angle Z
3	5.19615	0.71568	5.19615	0.71568	0.32476	$\bar{1}$.51156	0.43301	$\bar{1}$.63650	0.38490	$\bar{1}$.58534	120°
4	4.00000	0.60206	5.65685	0.75257	0.50000	$\bar{1}$.69897	1.00000	0.00000	0.35355	$\bar{1}$.54845	90°
5	3.63271	0.56023	5.87785	0.76921	0.59441	$\bar{1}$.77408	1.72048	0.23564	0.34026	$\bar{1}$.53181	72°
6	3.46410	0.53959	6.00000	0.77815	0.64951	$\bar{1}$.81259	2.59808	0.41465	0.33333	$\bar{1}$.52287	60°
7	3.37100	0.52775	6.07435	0.78350	0.68410	$\bar{1}$.83512	3.63393	0.56037	0.32925	$\bar{1}$.51753	51°26′
8	3.31371	0.52031	6.12294	0.78696	0.70710	$\bar{1}$.84948	4.82843	0.68380	0.32604	$\bar{1}$.51407	45°
9	3.27573	0.51530	6.15636	0.78932	0.72313	$\bar{1}$.85922	6.18182	0.79111	0.32486	$\bar{1}$.51170	40°
10	3.24920	0.51177	6.18034	0.79101	0.73473	$\bar{1}$.86612	7.69421	0.88616	0.32360	$\bar{1}$.51001	36°
11	3.22989	0.50918	6.19811	0.79225	0.74338	$\bar{1}$.87121	9.36566	0.97153	0.32267	$\bar{1}$.50877	32°44′
12	3.21539	0.50723	6.21166	0.79320	0.75000	$\bar{1}$.87506	11.1962	1.04906	0.32197	$\bar{1}$.50782	30°
13	3.20420	0.50572	6.22219	0.79394	0.75517	$\bar{1}$.87804	13.1858	1.12010	0.32143	$\bar{1}$.50708	27°42′
14	3.19543	0.50452	6.23062	0.79453	0.75929	$\bar{1}$.88041	15.3344	1.18566	0.32099	$\bar{1}$.50649	25°43′
15	3.18835	0.50356	6.23735	0.79500	0.76263	$\bar{1}$.88231	17.6424	1.24655	0.32064	$\bar{1}$.50603	24°
16	3.18260	0.50278	6.24289	0.79538	0.76536	$\bar{1}$.88387	20.1094	1.30339	0.32036	$\bar{1}$.50564	22°30′
17	3.17788	0.50213	6.24754	0.79570	0.76763	$\bar{1}$.88515	22.7353	1.35670	0.32012	$\bar{1}$.50532	21°11′
18	3.17389	0.50159	6.25133	0.79597	0.76954	$\bar{1}$.88623	25.5208	1.40689	0.31993	$\bar{1}$.50505	20°
19	3.17051	0.50113	6.25455	0.79619	0.77116	$\bar{1}$.88714	28.4654	1.45431	0.31976	$\bar{1}$.50483	18°57′
20	3.16769	0.50074	6.25738	0.79639	0.77254	$\bar{1}$.88792	31.5688	1.49925	0.31962	$\bar{1}$.50463	18°
21	3.16523	0.50040	6.25975	0.79655	0.77372	$\bar{1}$.88858	34.8316	1.54197	0.31950	$\bar{1}$.50447	17°9′
22	3.16317	0.50012	6.26195	0.79671	0.77476	$\bar{1}$.88916	38.2527	1.58266	0.31938	$\bar{1}$.50432	16°22′
23	3.16129	0.49986	6.26369	0.79683	0.77566	$\bar{1}$.88967	41.8342	1.62153	0.31930	$\bar{1}$.50420	15°39′
24	3.15966	0.49964	6.26526	0.79693	0.77645	$\bar{1}$.89011	45.5745	1.65872	0.31922	$\bar{1}$.50409	15°
25	3.15824	0.49944	6.26666	0.79703	0.77715	$\bar{1}$.89050	49.4738	1.69437	0.31914	$\bar{1}$.50399	14°24′

Contributed by W. L. Benitz, MACHINERY'S Data Sheet No. 105. Explanatory note: Page 38.

SQUARE AND HEXAGON NUT AND BOLT-HEAD DIMENSIONS



d	D	D'	d	D	D'	d	D	D'
1/4	0.2886	0.3535	1 1/4	1.4434	1.7677	2 5/16	2.6702	3.2703
9/32	0.3247	0.3977	1 9/32	1.4794	1.8119	2 3/8	2.7424	3.3587
5/16	0.3608	0.4419	1 5/16	1.5155	1.8561	2 7/16	2.8145	3.4471
11/32	0.3968	0.4861	1 11/32	1.5516	1.9003	2 1/2	2.8867	3.5355
3/8	0.4329	0.5303	1 3/8	1.5877	1.9445	2 9/16	2.9589	3.6239
13/32	0.4690	0.5745	1 13/32	1.6238	1.9887	2 5/8	3.0311	3.7123
7/16	0.5051	0.6187	1 7/16	1.6598	2.0329	2 11/16	3.1032	3.8007
15/32	0.5412	0.6629	1 15/32	1.6959	2.0771	2 3/4	3.1754	3.8891
1/2	0.5773	0.7071	1 1/2	1.7320	2.1213	2 13/16	3.2476	3.9794
17/32	0.6133	0.7513	1 17/32	1.7681	2.1655	2 7/8	3.3197	4.0658
9/16	0.6494	0.7955	1 9/16	1.8042	2.2097	2 15/16	3.3919	4.1542
19/32	0.6855	0.8397	1 19/32	1.8403	2.2539	3	3.4641	4.2426
5/8	0.7216	0.8839	1 5/8	1.8764	2.2981	3 1/16	3.5362	4.3310
21/32	0.7576	0.9281	1 21/32	1.9124	2.3423	3 1/8	3.6084	4.4194
11/16	0.7937	0.9723	1 11/16	1.9485	2.3865	3 3/16	3.6806	4.5078
23/32	0.8298	1.0164	1 23/32	1.9846	2.4306	3 1/4	3.7527	4.5962
3/4	0.8659	1.0606	1 3/4	2.0207	2.4708	3 5/16	3.8249	4.6846
25/32	0.9020	1.1048	1 25/32	2.0568	2.5190	3 3/8	3.8971	4.7729
13/16	0.9380	1.1490	1 13/16	2.0929	2.5632	3 7/16	3.9692	4.8613
27/32	0.9741	1.1932	1 27/32	2.1289	2.6074	3 1/2	4.0414	4.9497
7/8	1.0102	1.2374	1 7/8	2.1650	2.6516	3 9/16	4.1136	5.0381
29/32	1.0463	1.2816	1 29/32	2.2011	2.6958	3 5/8	4.1857	5.1265
15/16	1.0824	1.3258	1 15/16	2.2372	2.7400	3 11/16	4.2579	5.2149
31/32	1.1184	1.3700	1 31/32	2.2733	2.7842	3 3/4	4.3301	5.3033
1	1.1547	1.4142	2	2.3094	2.8284	3 13/16	4.4023	5.3917
1 1/32	1.1907	1.4584	2 1/32	2.3453	2.8726	3 1/8	4.4744	5.4801
1 1/16	1.2268	1.5026	2 1/16	2.3815	2.9168	3 15/16	4.5466	5.5684
1 3/32	1.2629	1.5468	2 3/32	2.4176	2.9610	4	4.6188	5.6568
1 1/8	1.2990	1.5910	2 1/8	2.4537	3.0052	4 1/8	4.7631	5.8336
1 5/32	1.3351	1.6352	2 5/32	2.4898	3.0494	4 1/4	4.9047	6.0104
1 3/16	1.3712	1.6793	2 3/16	2.5259	3.0936	4 3/8	5.0518	6.1872
1 7/32	1.4073	1.7235	2 1/4	2.5981	3.1820	4 1/2	5.1961	6.3639

Contributed by Howard D. Yoder, MACHINERY'S Data Sheet No. 84. Explanatory note: Page 39.

TABLE OF GEOMETRICAL PROGRESSION.

Ratio.	Increase in per cent.	1	2	3	4	5	6	7	8	9	10	11	12
1.10	10	1	1.10	1.21	1.33	1.46	1.61	1.77	1.95	2.14	2.36	2.59	2.85
1.15	15	1	1.15	1.32	1.52	1.75	2.01	2.31	2.66	3.06	3.52	4.05	4.65
1.20	20	1	1.20	1.44	1.73	2.07	2.49	2.99	3.58	4.30	5.16	6.19	7.43
1.25	25	1	1.25	1.56	1.95	2.44	3.05	3.81	4.77	5.96	7.45	9.31	11.64
1.30	30	1	1.30	1.69	2.20	2.86	3.71	4.83	6.27	8.16	10.61	13.79	17.92
1.35	35	1	1.35	1.82	2.46	3.32	4.48	6.05	8.17	11.03	14.88	20.09	27.14
1.40	40	1	1.40	1.96	2.74	3.84	5.38	7.53	10.54	14.76	20.66	28.93	40.50
1.45	45	1	1.45	2.10	3.05	4.42	6.41	9.29	13.48	19.54	28.33	41.09	59.57
1.50	50	1	1.50	2.25	3.38	5.06	7.59	11.39	17.09	25.63	38.44	57.67	86.50
1.55	55	1	1.55	2.40	3.72	5.77	8.95	13.87	21.49	33.32	51.64	80.04	124.10
1.60	60	1	1.60	2.56	4.10	6.55	10.49	16.78	26.84	42.95	68.72	110.00	.....
1.65	65	1	1.65	2.72	4.49	7.41	12.23	20.18	33.30	54.94	90.65	149.60	.....
1.70	70	1	1.70	2.89	4.91	8.35	14.20	24.14	41.03	69.75	118.60	.....	.....
1.75	75	1	1.75	3.06	5.36	9.38	16.41	28.72	50.27	87.96	153.90	.....	.....
1.80	80	1	1.80	3.24	5.83	10.50	18.90	34.01	61.22	110.20	.....	.....	.....
1.85	85	1	1.85	3.42	6.33	11.71	21.67	40.09	74.17	137.20	.....	.....	.....
1.90	90	1	1.90	3.61	6.86	13.03	24.76	47.05	89.39	169.80	.....	.....	.....
1.95	95	1	1.95	3.80	7.41	14.46	28.20	54.98	107.20	.....	.....	.....	.....
2.00	100	1	2.00	4.00	8.00	16.00	32.00	64.00	128.00	.....	.....	.....	.....

		13	14	15	16	17	18	19	20	21	22	23	24
1.10	10	3.14	3.45	3.80	4.18	4.60	5.05	5.56	6.11	6.73	7.40	8.14	8.95
1.15	15	5.35	6.15	7.08	8.14	9.36	10.76	12.38	14.23	16.37	18.82	21.65	24.89
1.20	20	8.92	10.70	12.84	15.41	18.49	22.19	26.62	31.95	38.34	46.01	55.21	66.25
1.25	25	14.55	18.19	22.74	28.42	35.53	44.41	55.51	69.39	86.74	108.40	....	....
1.30	30	23.30	30.29	39.37	51.19	66.54	86.50	112.50	....	....	.....	....	....
1.35	35	36.64	49.47	66.78	90.37	121.70	....	.....	....	....	.....	....	....
1.40	40	56.69	79.37	111.10	....	.....	....	.....	....	....	.....	....	....
1.45	45	86.38	125.30	.....	....	.....	....	.....	....	....	.....	....	....
1.50	50	129.80	.....	.....	....	.....	....	.....	....	....	.....	....	....

GEOMETRICAL PROGRESSIONS FOR SPINDLE SPEEDS—I

Number of Speed Changes Required	Percentage of decrease of consecutive speeds.												
	15	16	17	18	19	20	21	22	23	24	25	26	27
1	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
2	850	840	830	820	810	800	790	780	770	760	750	740	730
3	722	706	689	672	656	640	624	608	593	578	562	548	533
4	614	593	572	551	531	512	493	474	456	439	421	405	389
5	522	498	474	452	430	410	389	370	351	333	316	300	284
6	444	418	394	371	349	328	308	289	271	253	237	222	207
7	377	351	327	304	282	262	243	225	208	193	178	163	151
8	320	295	271	249	229	210	192	176	160	146	133	121	110
9	272	248	225	204	185	168	152	137	123	111	100	89.4	81
10	231	208	187	167	150	134	120	107	95	84.5	75	66.1	59
11	197	175	155	137	121	107	94.6	83.3	73.2	64.2	56.2	48.9	43
12	167	147	128	113	98.4	86	74.7	65	56.4	48.8	42.2	36.2	31.3
13	142	123	107	92.4	79.7	68.7	59.1	50.7	43.4	37.1	31.6	26.8	22.8
14	121	103	88.5	75.7	64.5	55	46.6	39.5	33.4	28.2	23.7	19.8	16.6
15	103	87	73.4	62.1	52.3	44	36.9	30.8	25.7	21.4	17.8	14.6	12.15
16	87.3	73	60.9	50.9	42.3	35.2	28.1	24	19.8	16.3	13.3	10.8	8.87
17	74.2	61.3	50.5	41.7	34.2	28.16	23	18.7	15.2	12.4	10	8.0	6.47
18	63	51.5	41.5	34.2	27.7	22.5	18.2	14.6	11.7	9.4	7.5	5.9	4.7
19	53.6	43.2	34.8	28	22.4	18	14.3	11.4	9.0	7.14	5.6	4.4	3.44
20	45.5	36.3	28.9	23	18.1	14.9	11.3	8.9	6.9	5.42	4.2	3.25	2.51
21	38.7	30.5	24	18.9	14.7	11.5	8.9	6.94	5.4	4.12	3.2	2.4	1.53
22	32.9	25.6	19.9	15.5	11.9	9.2	7.1	5.4	4.1	3.13	2.4	1.78	1.33
23	27.9	21.5	16.5	12.7	9.6	7.37	5.6	4.22	3.2	2.38	1.8	1.31	0.97
24	23.7	18	13.7	10.4	7.8	5.9	4.4	3.29	2.46	1.81	1.3	0.97	

Contributed by Albert Clegg, MACHINERY's Data Sheet No. 105. Explanatory note: Page 39.

GEOMETRICAL PROGRESSIONS FOR SPINDLE SPEEDS—II

Number of Speed Changes Required	Percentage of decrease of consecutive speeds. (Continued)												
	28	29	30	32	34	36	38	40	42	44	46	48	50
1	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
2	720	710	700	680	660	640	620	600	580	560	540	520	500
3	518	504	490	462	436	410	385	360	337	314	292	270	250
4	373	358	343	31.5	288	263	238	216	195	176	158	141	125
5	269	254	240	214	190	169	148	129	113	98	85	73.3	62.5
6	193	180	168	146	120	108	91.7	77.5	65.5	54.8	46	38.2	31.25
7	139	128	118	99	83	69.2	56.8	46.5	38	30.7	24.8	19.9	15.62
8	100	91	82.3	68.3	54.8	44.4	35.2	27.9	22.1	17.2	13.4	10.3	7.81
9	72.2	64.5	57.6	46.5	36.2	28.4	21.8	16.8	12.8	9.6	7.23	5.37	3.91
10	52.4	45.8	40.3	31.5	23.8	18.2	13.5	10.0	7.42	5.38	3.9	2.8	1.95
11	37.4	32.5	28.2	21.4	15.8	11.63	8.38	6.0	4.3	3.0	2.1	1.45	0.97
12	26.9	23	19.8	14.5	10.4	7.46	5.2	3.6	2.5	1.68	1.13	0.75	0.475
13	19.4	16.3	13.8	9.85	6.9	4.78	3.22	2.16	1.45	0.94			
14	13.9	11.6	9.7	6.7	4.55	3.06	2.0	1.29	0.84				
15	10	8.2	6.8	4.55	3.0	1.96	1.24	0.77					
16	7.23	5.8	4.7	3.09	1.99	1.25	0.77						
17	5.21	4.14	3.32	2.1	1.32	0.8							
18	3.75	2.94	2.32	1.43	0.87								
19	2.7	2.08	1.63	0.97									
20	1.94	1.48	1.14										
21	1.39	1.05											
22	1.00												
23													
24													

Example: A lathe spindle requires 18 speed changes, from 6 to 250 R.P.M. Find the speeds in geometrical progression.

$\frac{1000}{250} = 4$ *(ratio between highest R.P.M. in table and highest R.P.M. in example);* $4 \times 6 = 24$. *Follow the horizontal line from figure 18 in the left-hand column to the figure nearest to 24 in this line. This figure is 22.5, in the 20 per cent column. The speeds required are the values given in this column opposite figures 1 to 18, divided by 4. Thus 250, 200, 160, 128, etc., are the speeds wanted.*

Decimals of a Minute for Each Quarter Second

On pages 30 and 31 a table is given of the equivalent decimals of a minute corresponding to each quarter of a second from 1/4 to 60 seconds. Of course, this table can also be used for finding the equivalent decimals of a degree corresponding to fractions of a minute. For example, 9 seconds equals 0.15 minute, and 9 minutes equals 0.15 degree, as found in the third and fourth columns on page 30. [MACHINERY'S Reference Series No. 18, Shop Arithmetic for the Machinist, Third Edition, Chapter X.]

Computation Table for Regular Polygons

The object of the two tables on pages 32 and 33, relating to regular polygons, is to simplify the necessary calculations for any required dimensions of polygons. If the area, or any dimension of a regular polygon of a specified number of sides, is given, all the other dimensions are thereby determined in amount, and the proportion which the known part bears to each of the unknown parts will vary with the number of sides of the polygon. A knowledge of trigonometry is necessary for determining the required proportions of these parts, and the accompanying tables were prepared not only to facilitate the trigonometrical work, but, at the same time, to simplify the process down to plain multiplication, or where the area is the given dimension, to the extraction of a square root. The case in which the number of sides is required cannot be directly solved by elementary trigonometry, but is easily found by means of the tables.

The table on page 32 gives the symbols by which the various parts are designated, as well as all the necessary formulas needed for any computation. In addition to the symbols indicated at the top of this table, there appear the letters B, F, H, K and M, which are calculated in the table on page 33 for the proper value of N, the number of sides of the polygon, which heads the first column. These letters represent the following quantities:

$$F = N \tan \frac{180^\circ}{N}; \quad M = 2N \sin \frac{180^\circ}{N};$$

$$B = \frac{1}{N} \operatorname{cosec} \frac{180^\circ}{N}; \quad K = \frac{N}{4} \cot \frac{180^\circ}{N};$$

$$H = \frac{N}{8} \sin \frac{360^\circ}{N}.$$

The logarithms of these quantities are given in each succeeding column for convenience in logarithmic calculation. The use of the tables may be partly illustrated by the following simple applications.

Example 1.—What size octagon steel will have a sectional area of 2 square inches?

We know $A = 2$, $N = 8$, and must find d. In the column headed A, on page 32, we run down the column until we find d on the left-hand side of the equation, giving the formula $d = \frac{4\sqrt{AK}}{N}$. The value of K is taken from page 33, in the line beginning with $N = 8$, giving us $K = 4.82843$. Substituting these values of A, K, and N in the formula gives:

$$d = \frac{4\sqrt{2 \times 4.82843}}{8} = \frac{\sqrt{9.65686}}{2} =$$

1.554 inch, which is the proper distance across the flats to give a sectional area of 2 square inches.

Example 2.—What is the area of a segment of a circle, 66 inches in diameter, lying between the circumference and one side of an inscribed polygon of 5 sides? The area required, which is similar to the shaded area in the figure on page 32, will be one-fifth of the difference in areas between the circle and the inscribed polygon. Knowing D, we find the area of the polygon from the formula $A = HD^2$, using the value of

H for $N = 5$, which is 0.59441. The area of the polygon is then $A = (66)^2 \times 0.59441 = 2589.3$ square inches. The area of the circle is $\frac{\pi}{4} (66)^2 = 3421.2$ square inches. Taking one-fifth of the difference we have

$$\text{Area of segment} = \frac{3421.2 - 2589.3}{5} =$$

166.38 square inches.

These are only a few of the many applications that may be made of the tables, and as indicated by the number of formulas, there may be twenty different problems stated, all of which are taken care of. These formulas and tables are also adapted to the solution of isosceles triangles in which the vertical angle approximates that in column Z, on page 33: [MACHINERY'S Reference Series No. 52, Advanced Shop Arithmetic for the Machinist, page 33, Regular Polygons.]

Square and Hexagon Nut and Bolt-head Dimensions

The table on page 34 gives the relation between the distance "across flats" and the distance "across corners" of squares and hexagons, and is especially applicable where it is required to find the distance across corners of square and hexagon bolt-heads and nuts, when the distance across flats is known. As an example, if the distance across flats of a hexagon nut is 1 7/8 inch, then the distance across corners is 2.165 inches. The distance across corners of a square bolt-head, 1 7/8 inch across flats, would be 2.6516 inches. Of course, in practical calculations it is unnecessary to use more than two decimals, but the table has been calculated to four decimals in order to make it useful in cases where calculations other than of nut and bolt-head dimensions are made, and where a higher degree of accuracy is required. [MACHINERY'S Reference Series No. 52, Advanced Shop Arithmetic for the Machinist, page 33, Regular Polygons.]

Table of Geometrical Progression

On page 35 a table is given of geometrical progression where the increase of successive terms, in per cent, varies from 10 to 100. The columns 1, 2, 3, 4, etc., give the values of the respective terms in the progression when the first term is 1. For example, if the increase in per cent is 15, and the first term in the progression is 1, then the eighth term in this geometrical progression would be 2.66. If the first term is any other value than 1, the value of the other terms may be found by multiplying the various tabulated values by the value of the first term. This table will be found especially useful in designing the drives for machine tools. The tables on pages 36 and 37 are intended for the same purpose, an extended explanation of the use of these tables being given in the following.

Calculations for Cone Drive and Back-gear Design

One of the first problems met with in the design of a machine tool is that of determining suitable spindle speeds. In a correct design the various speeds must have a fixed relation to each other, this relation being that of geometrical progression, and the problem is to find the intermediate speeds, the various diameters of the cone steps, and the back-gear ratio, when the slowest and fastest speeds and the diameter of one of the cone steps are given. The calculations for finding these speeds are rather lengthy, and the machine designer will no doubt appreciate any method which permits him to save time and work in the laying out of his drive. The tables given on pages 36 and 37 will be found to be very valuable for this kind of work. These tables give a number of geometrical progressions ranging from speeds decreasing by 15 per cent to speeds decreasing by 50 per cent.

The simplest way to describe the advantages of this table will be by a prac-

tical example. Assume that the spindle of a lathe requires 18 speed changes, varying from 6 to 250 R.P.M., that the cone has three steps, the largest step being 15 inches in diameter, that the lathe is double back-geared, and that a two-speed counter-shaft is provided. The questions to be answered are then: What are the intermediate speeds between 6 and 250 R. P. M.? What are the diameters of the two remaining cone steps? What are the back-gear ratios? and what should be the counter-shaft speeds?

If we turn to the tables we will find that the maximum speed in every case is given as 1000, which, in the case of the lathe we are to design, is four times greater than the maximum speed of the spindle. In order to reduce the figures given in the table to correspond with those of our example, we must divide the speeds given in the table by 4; $(1000 \div 4 = 250)$; our slowest speed, given as 6 R. P. M., will then correspond to a speed given in the table equal to 24 R. P. M.; $(24 \div 4 = 6)$. It will be seen that 24 and 1000 are in exactly the same ratio as 6 and 250. The number of spindle speeds being 18, we now follow the horizontal line from the figure 18 in the left-hand column of the table, until we reach the number nearest to 24, the number in this case being 22.5 in the 20 per cent column. The figures in this column, divided by 4, will give us the range of the speeds desired, these speeds being as follows:

RANGE OF SPEEDS

250	65.5	17.17
200	52.5	13.7
160	42	11
128	33.5	8.8
102.5	26.75	7.04
82	21.5	5.62

The speed ratio for the first back-gears is obtained by dividing 250, the fastest open belt speed, by 65.5, the fastest first back-gear speed; and the second back-gear ratio is found by dividing 250, the fastest open belt speed, by 17.17, the fastest second back-gear speed. These two ratios are then found to be 3.82 to 1, and 14.6 to 1, respectively. The counter-shaft speeds will be found to be 200 and 102.5 (the speeds of the middle cone steps), if consecutive spindle speeds are obtained by moving the belt from one step on the cone to another; but if consecutive speeds are obtained by shifting the counter-shaft, then this latter would be required to run at 160 and 128 R. P. M., which would then be the speeds of the middle cone steps. The diameter of the smallest cone step, if consecutive speeds are obtained by changing the counter-shaft speed, will equal $\frac{15 \times 160}{250}$ $= 9.6$ inches. The diameter of the middle step would then be 12.3 inches. It will be seen that the table diminishes the actual work required for calculating geometrical speed progressions to a considerable extent, and the various quantities to be determined can be found with very little arithmetical work.

If it be required to lay out a drive with four cone steps instead of three, the calculations for the counter-shaft speeds, of course, become a little more complicated, as there is then no middle cone step, but the use of the table for finding spindle speeds and back-gear ratios will eliminate a great amount of work even in this case. [MACHINERY, March, 1909, Geometrical Progression for Spindle Speeds; MACHINERY'S Reference Series No. 16, Machine Tool Drives.]

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